T. Hirashima et al. (Eds.) (2011). Proceedings of the 19th International Conference on Computers in Education. Chiang Mai, Thailand: Asia-Pacific Society for Computers in Education

A Computer-Assisted Environment for Learning Function Finding

Wing-Kwong Wong [a], Kai-Jie Jhuang [b], Yunn-Wen Lien [c], Chao-Jung Wu [d]
[a]*National Yunlin University of Science & Technology, Department of Electronic Engineering, Douliou, Yunlin, Taiwan*
[b]*National Yunlin University of Science & Technology, Graduate School of Computer Science and Information Engineering, Douliou, Yunlin, Taiwan*
[c]*National Taiwan University, Department of Psychology, Taipei, Taiwan*
[d]*National Taiwan Normal University, Department of Educational Psychology and Counseling, Taipei, Taiwan*
{ [a]wongwk, [b]g9717713}@yuntech.edu.tw
[c]ywlien@ntu.edu.tw
[d]cjwu@ntnu.edu.tw

Abstract: Inductive reasoning is an important domain-general skill for learning science and math, etc. Unfortunately, previous studies have pointed out that students still have many weaknesses, when they do the tasks of inductive reasoning. A web-based environment is designed to train their skills of inductive reasoning. The pedagogy of self-directed experimentation is adopted in the learning environment, in which learners are asked to express one variable as a function of another variable. Two input interfaces are used for the users to input expressions. The control group uses a traditional tool of expressing a dependent variable as a function of an independent variable. The experimental group uses a tool that can generate a new term by addition, subtraction or multiplication of two existing terms of expressions. Some interesting results are reported.

Keywords: Inquiry-based learning, inductive skills, pattern-finding, hypothesis generation, function-finding.

1. Introduction

The ability of doing tasks similar to those of scientific invention is important to learners in all ages and is noted in almost every education project. But how to teach these skills to learners still remains a mysterious problem. In traditional science laboratory, learners in high schools or university are asked to follow the steps in lab manuals. They are expected to generate data that are predicted by theory. When learners get unexpected data in an experiment, they might change the data just to fit the theoretical prediction.

To take a different approach, we propose, following Bruner [1][2], that the students can learn by active discovery. In addition to learning domain knowledge, students should also learn to explore and generate hypotheses. With curiosity and excitement, students should learn to discover patterns in data, resulting in their active construction of knowledge. Hopefully, the knowledge will be engraved in their mind so deeply that they can readily transfer the acquired knowledge to other domains upon situations that call for the application of such knowledge. In this way, they became more confident in their own ability. This way of learning could be better than mechanical rote learning and repeated drill, which often result in the loss of interest in learning and in the failure of applying the memorized rules to solve real-world problems.

In a study to investigate the process of inductive reasoning [3] in function-finding tasks, Haverty et al. [4] proposed several stages of inductive reasoning activity: data gathering, pattern finding, and hypothesis generation. Data gathering is defined to include both data collection activities and the organization and representation. Pattern finding includes activities of investigation and analysis of the data collected. Hypothesis generation encompasses the activities of constructing, proposing, and testing hypotheses.

Haverty et al. [4] found that when the subjects were asked to find a mathematical formula to relate two terms x and y (whose data are given), there are three common strategies in generating successful hypotheses. The first one is local hypothesis strategy. In local hypothesis strategy, subjects formed a local hypothesis from a single instance of data to test whether the local hypothesis holds for other instances. Before finding a hypothesis that works on all instances, they may have to generate many local hypotheses. Nevertheless, this strategy is useful in finding the elements of the global, target law.

Another strategy that the subjects employed successfully to solve the problems was called the pursuit strategy. In this strategy, subjects detect a pattern in a quantity q, and then use q as a subgoal to understanding y. The subjects then tried to express q in terms of x, and composed expression into a global formula. The third strategy is recursion. This only worked when the subjects had rich theoretical insights to the idea of recursion.

A computer-assisted learning environment called InduLab is designed to help students improve their skills of inductive inferencing that are often used in inquiry-based learning [5]. In inquiry-based learning, three stages of inductive reasoning can be identified: (1) data gathering; (2) hypothesis generation; (3) hypothesis evaluation [6]. Some tools to help students in these stages are provided in the environment. In this study, the task of inquiry in simplified by providing all collected data of two given variables. Subjects are asked to generate hypotheses that express a variable as functions of another variable.

We want to examine the effects of two methods of entering an expression on the strategies of hypothesis generation in our computer-assisted learning environment. One input interface is to input a formula involving only one variable like a traditional calculator. Another input interface is to generate a new term by addition, subtraction or multiplication between two terms. Once a new term is entered, it can be reused in the next round of inputting a new term. This idea is motivated by Haverty's findings on the pursuit strategy. A major difference between Haverty's study and this study is that Haverty's subjects did not use any computer tools.

2. Experiment Design

A pilot experiment was designed to evaluate InduLab's scaffolding of two different input methods of expression. Six university students were asked to use InduLab for a function-finding task. They were divided into two groups, the control group and the treatment group [7][8].

The subjects of both groups read a figure that showed how the two terms of variables, namely n and m, were obtained. n was the serial number of the data set of m counting from

1 to 5. m was the number of balls in the n^{th} pattern. The data were (1,1), (2,5), (3,13), (4,25) and (5,41).

Figure 1 is the interface of inputting a function of n for the control group, similar to that of a calculator. The subjects can enter digits, variable n, and operators of addition, subtraction, and multiplication to input a formula. In the figure, the subject enters n*n+(n-1)*(n-1).

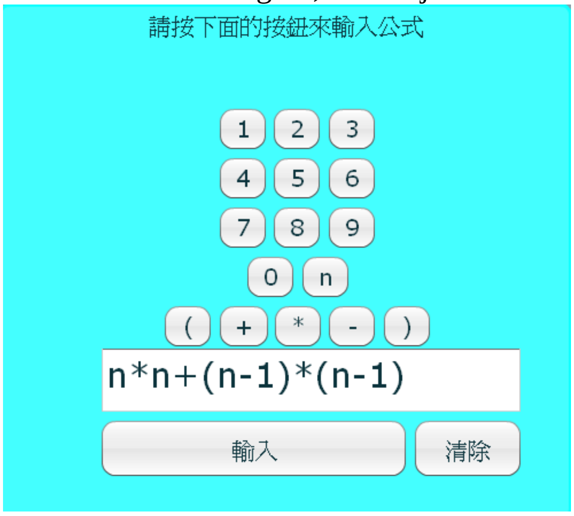

Figure 1: Interface of entering formulae for the control group

Figure 2 is the interface of entering experssion for the treatment group. Subjects are required to use two operands and one operator to generate a new hypothesis. Operands can be a number and any exisiting term, and the operators are +, -, *. For example, they can input n-1 or n*n as new terms. Before they can enter $(n-1)^2$, they have to generate the term n-1 first. Then the operand menu will show the term n-1. So one possible sequence of generating terms in order to produce the formula $2n^2-2n+1$ is: n-1, n*n, (n-1)*(n-1), $n^2+(n^2-2n+1)$.

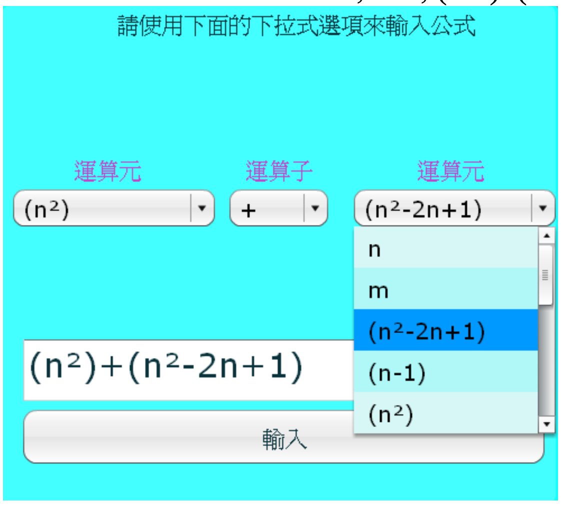

Figure 2: The interface of entering formulae for the treatment group

After the user enters a new hypothesis of expression, the system will generate the values of the expression for all the data (Table 1). Also, the algebraic expression will be expanded automatically by the system. For example, when a subject inputs (n-1)*(n-1), the expanded expression is n^2-2n+1. All expressions will be shown on a table. Since space is limited, the table only shows a limited no. of columns. But the user can choose which columns to show with a tool in the interface.

Table 1. Each column is an expression input by user and its function values from 1 to 5

編號(n)	總數(m)	n^2	n-1	n^2-2n+1	$2n^2$-2n+1
1	1	1	0	0	1
2	5	4	1	1	5
3	13	9	2	4	13
4	25	16	3	9	25
5	41	25	4	16	41

For the treatment group, subject can choose m as an operand when forming a new term. For example, they can input $m\text{-}n^2$ as shown in Table 2. Then the data of $m\text{-}n^2$ (0, 1, 4, 9, 16) are clearly square numbers. This data sequence can also be obtained by the new term $n^2\text{-}2n+1$. Then the subject should be able to infer that $m\text{-}n^2 = n^2\text{-}2n+1$, meaning that $m = 2n^2\text{-}2n+1$, which is the target formula.

Table 2. Two columns of identical values indicates the discovery of a target function

編號(n)	總數(m)	n-1	n^2	m-n^2	n^2-2n+1
1	1	0	1	0	0
2	5	1	4	1	1
3	13	2	9	4	4
4	25	3	16	9	9
5	41	4	25	16	16

3. Results from Experiment

In an experiment, the subjects in the control group and the treatment groups were asked to do the above function-finding task. The target formula was $2n^2\text{-}2n+1$. After the subjects in both groups finished their tasks, the data of their hypotheses were analyzed. The results are summarized in this section.

The three subjects in the control group started from simple functions of n to those of n^2 with larger coefficients gradually. At the end, they produced 42, 23 and 24 hypotheses. Only the third subject found the target formula. In the experimental group, all three subjects seemed to be using the local hypothesis strategy from time to time. They produced 7, 12, and 30 hypotheses. All three found the target formula. The first subject used the recursion strategy and found the target formulae with only seven hypotheses.

The most striking result of this experiment was that no one in the treatment group used the dependent variable m to form a hypothetical expression. This was different from Haverty's findings, where many subjects used m in their hypotheses.

4. Conclusion and Discussion

This study provided an E-learning environment to train students' skills of inductive reasoning for a task of function finding. We want to examine the effects of two different input methods of expressions on the subjects' strategies of hypothesis generation. The subjects in the control group used a traditional calculator interface to input a function of one single variable. The subjects in the treatment group used an interface that generates a new term by addition, subtraction, or multiplication between two existing terms. When the new term was generated, it can be used in the next round of term generation.

One finding of this pilot study was that the subjects in the treatement group tended to use the local hypothesis strategy from time to time. In the control group, subjects tended to enumerate formula with increasing complexity indicated by increasing degree and larger coefficients systematically. While systematic enumeration did not require many resources and was easy to do, this strategy provided few insights to finding the target formula. Moreover, if the problem involves a term of higher degree and thus becomes more complex, the search space gets much bigger and systematic enumeration might take too long to be an effective strategy. Future study needs to guide the subjects to avoid this shallow-thinking strategy.

A surprising result was that no subjects input any formula involving the dependent variable m when generating hypotheses. In future studies, more guidance should be provided for the subjects to try to involve m in the new term to be generated. This might lead to the use of the pursuit strategy, or to shorten the list of generated hypotheses and make the target finding process more efficient.

It seemed that the formula-input interface for the treatment group might be more effective in finding the target formula. Perhaps it made better use of old hypotheses by requiring that an earlier hypothesis must be used. The input interface for the control group did not force the subject to consider an earlier hypothesis. While this is an encouraging result, this deserves more investigation before any definitive conclusion can be drawn.

Acknowledgement
This study is supported by the National Science Council of Taiwan under contract NSC 98-2511-S-224-004-MY2.

References

[1] Bruner, J. S. (1960). The process of education. New York: Vintage Books.
[2] Bruner, J. S. (1961). Harvard Educational Review. 31, 21-32.
[3] Kinshuk, Taiyu Lin, Paul McNab. (2004). Modelling Inductive Reasoning Ability for Adaptive Virtual Learning Environment. In CELDA, 343-349.
[4] Haverty, L. A., Koedinger, K. R., Klahr, D. , & Alibali, M. W. (2000). Solving inductive reasoning problems in mathematics: Not-so-trivial pursuit. Cognitive Science, 24(2), 249-298.
[5] 黃永廣、連韻文、吳昭容、鄭英豪，2008，培養學童科學發現與創造力的數位學習環境之研究。數位學習國家型科技計畫結案報告。
[6] 陳亦媛，2003，兒童歸納推理能力探究-影響兒童證據與假設協調能力的可能原因與解決之道。台灣大學。
[7] 戴錦秀，2002，國小五年級學生使用電腦軟體GSP學習三角形面積成效之研究。高雄師師範大學。
[8] Wu, C. J., Wong, W. K., Cheng, Y. H. & Lien, Y. W. (2006). Generating and evaluating geometry conjectures with self-directed experiments. In Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education, Prague, Charles University.