NAC2005
NSTDA Annual Conference
S&T in Thailand : Towards the Molecular Economy

ECTI-21
NECTEC

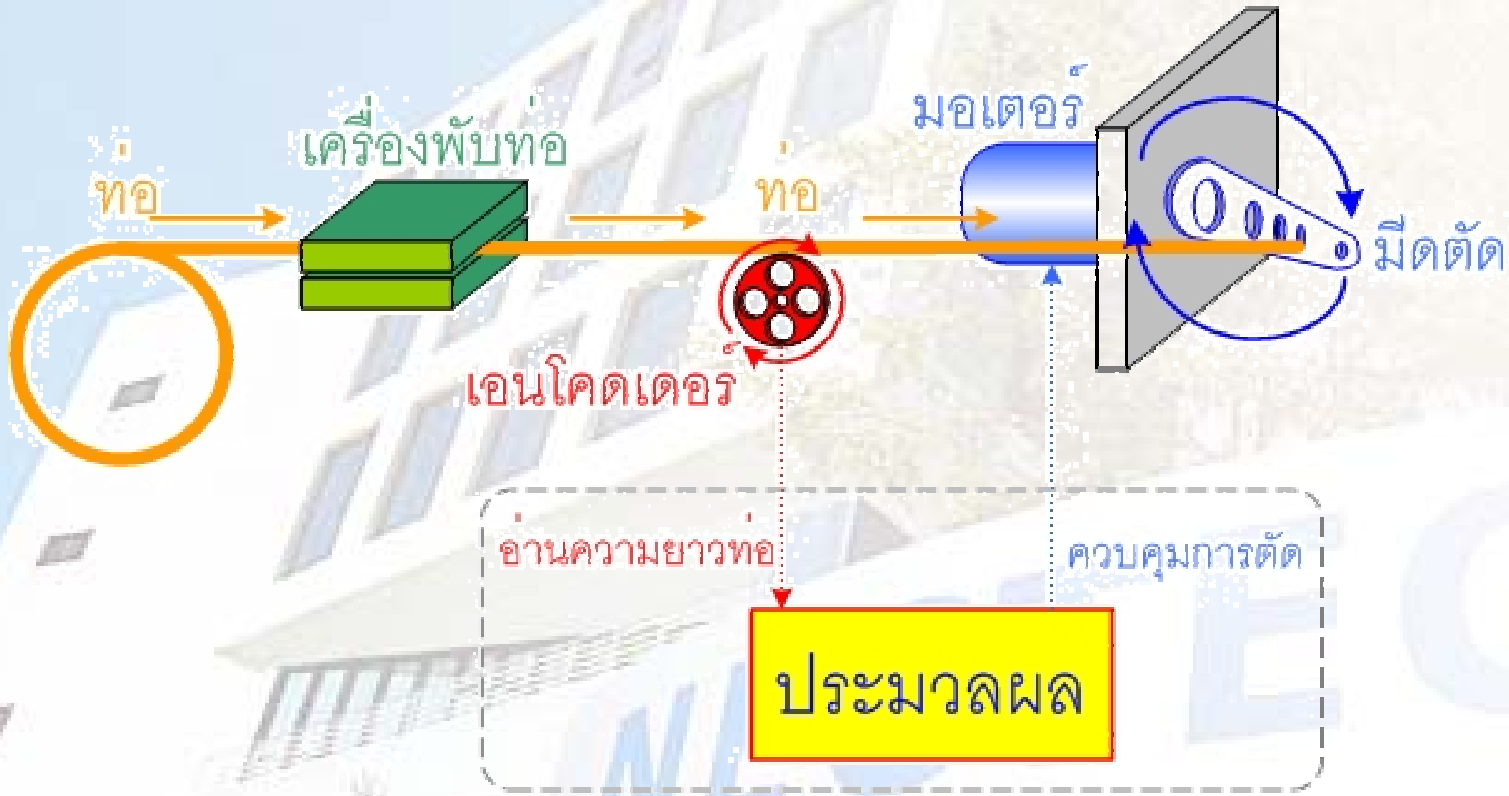
การนำตัวสังเกตทำนายค่าไปใช้บนตัวประมวลผลสัญญาณดิจิตอลแบบจำนวนเต็ม

Implementation of the Predictive Observer on a Fixed-point DSP

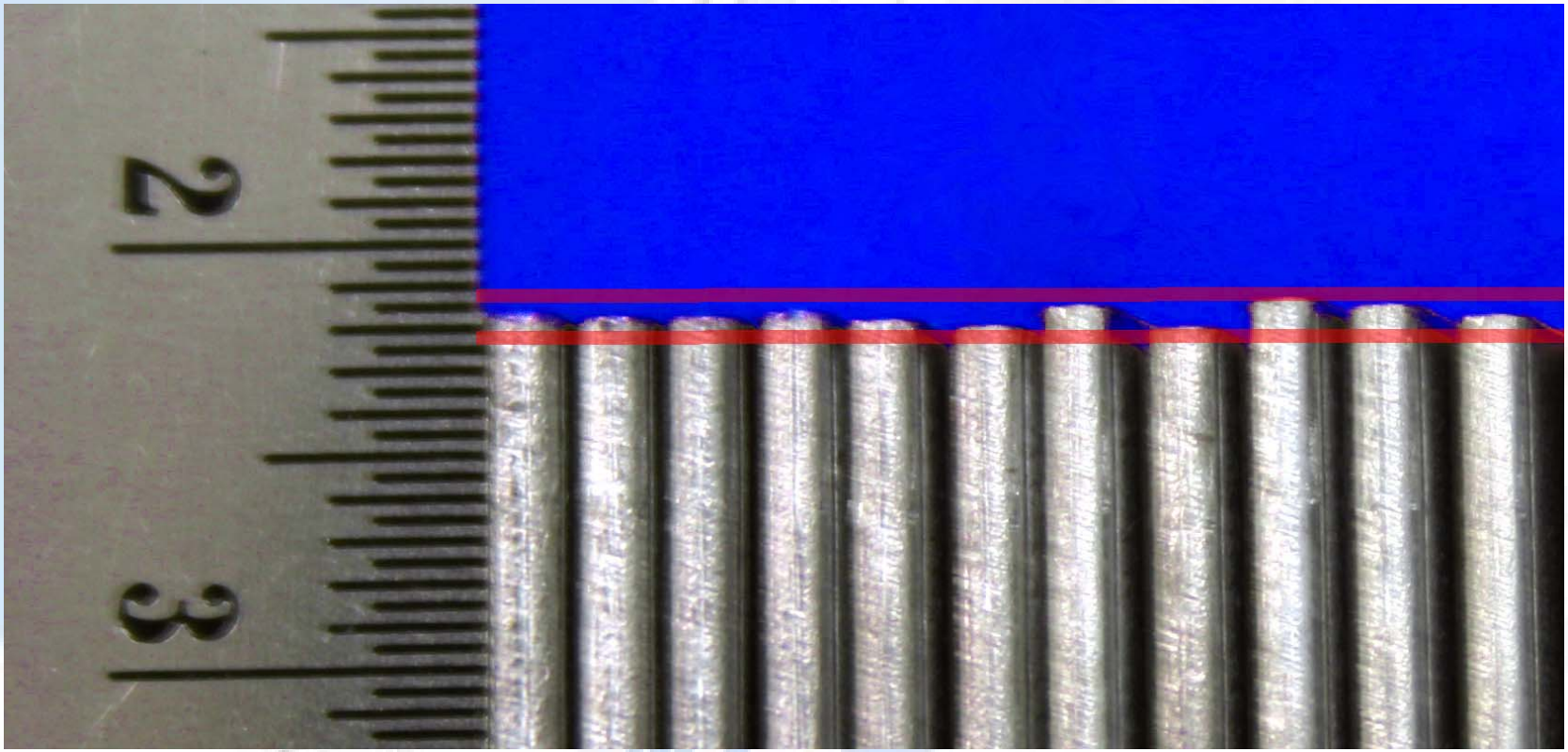
ทรงกรด ธิราชัย

งานวิจัยการผลิตอัตโนมัติ ฝ่ายวิจัยและพัฒนาเทคโนโลยีอิเล็กทรอนิกส์
ศูนย์เทคโนโลยีอิเล็กทรอนิกส์และคอมพิวเตอร์แห่งชาติ

ระบบเครื่องตัดท่อหม้อน้ำ



Target

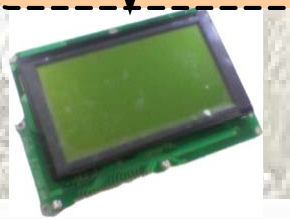


ท่อที่ตัดมีความผิดพลาดได้ไม่เกิน +/- 0.5 มิลลิเมตร

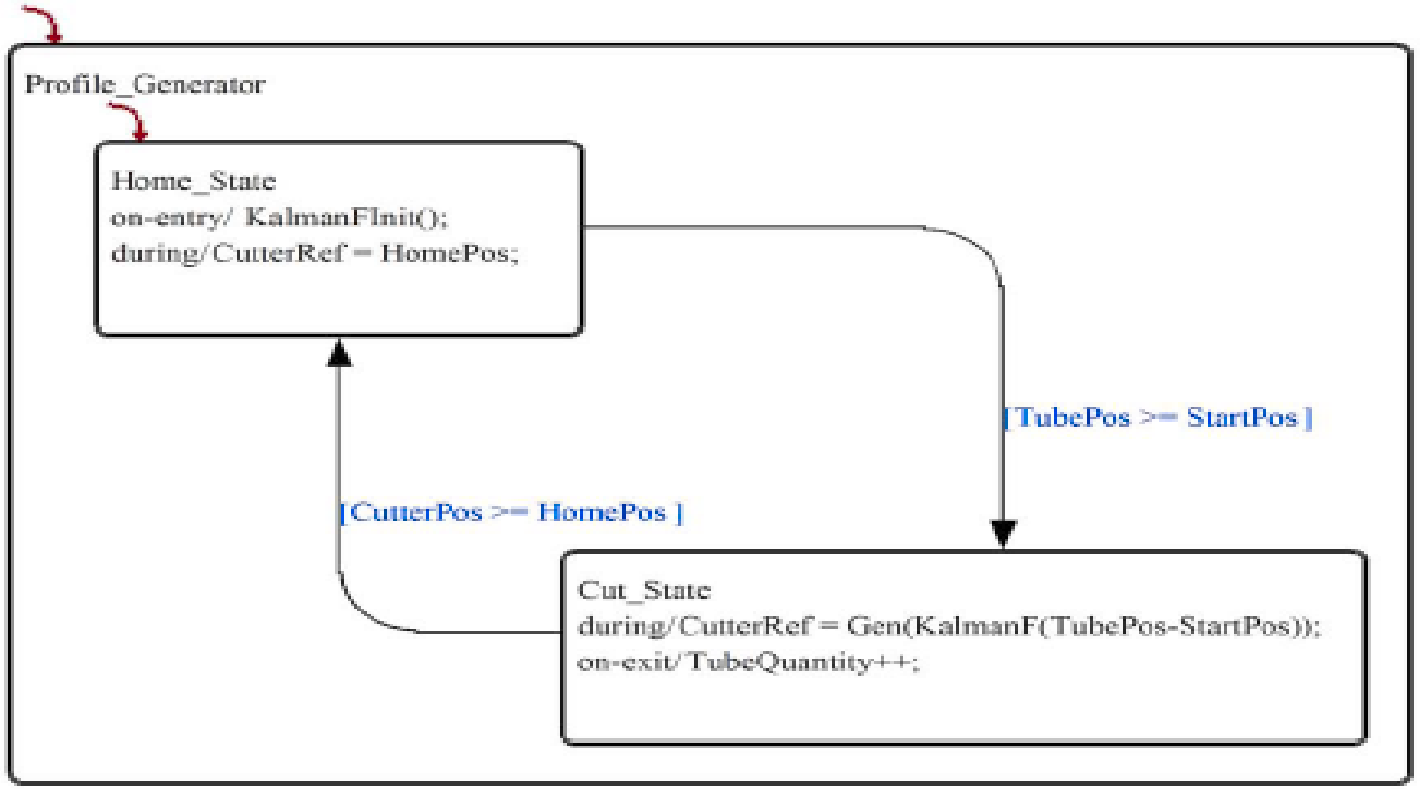
VDO

Solution

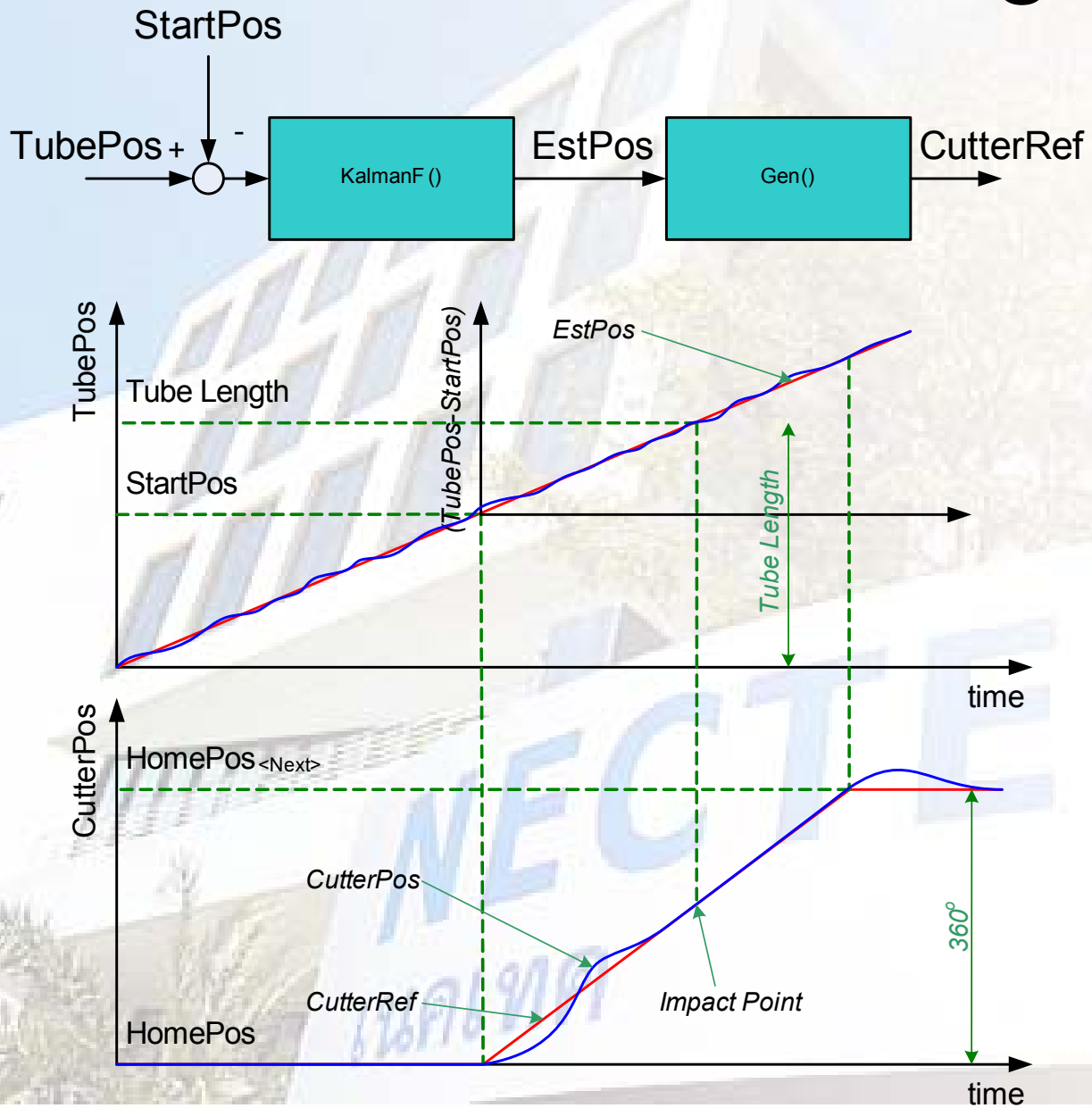
Encoder
Motor
Encoder Reader
Profile Generator
+
-
Controller
Motor Drive
Encoder Reader
Memory
Supervisor
I/O



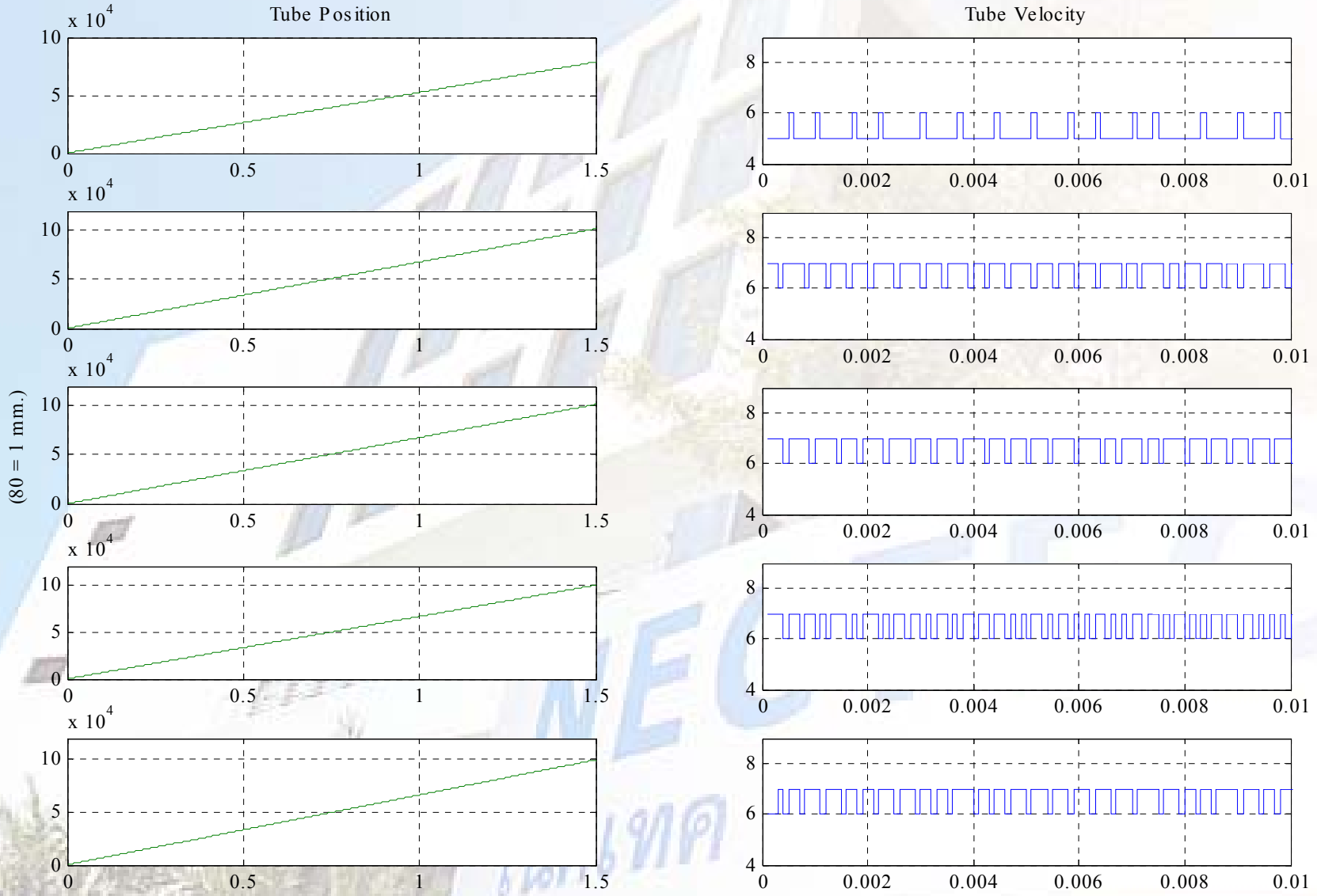
Profile Generator



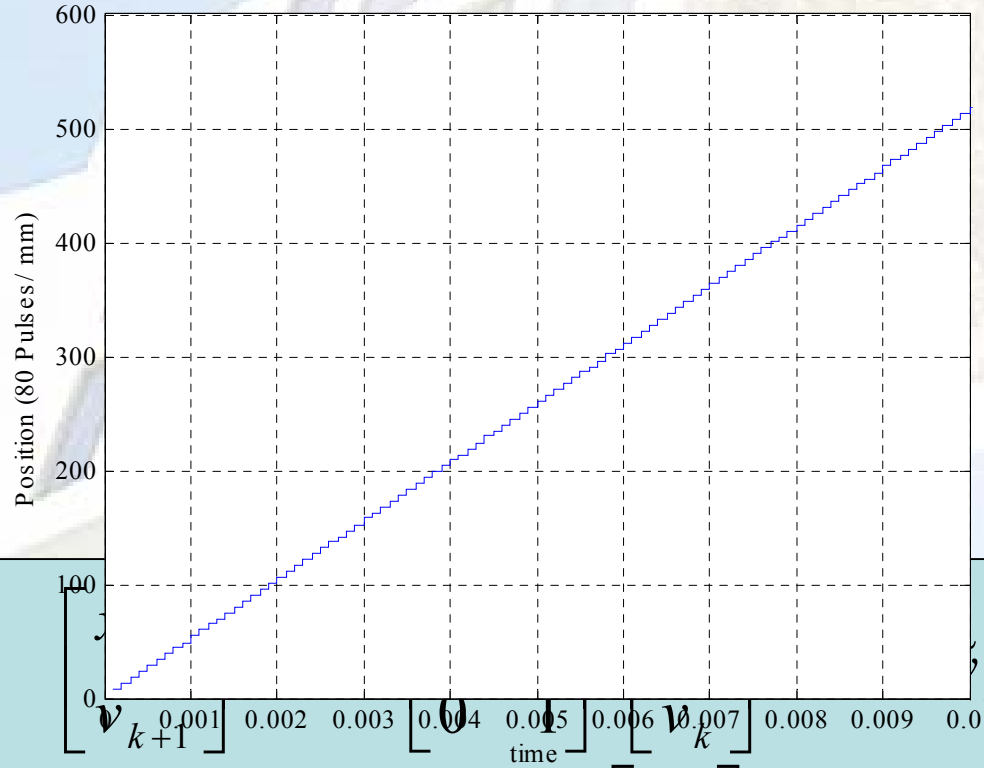
Profile Generator Design



Signal from Encoder



Model of tube encoder signal



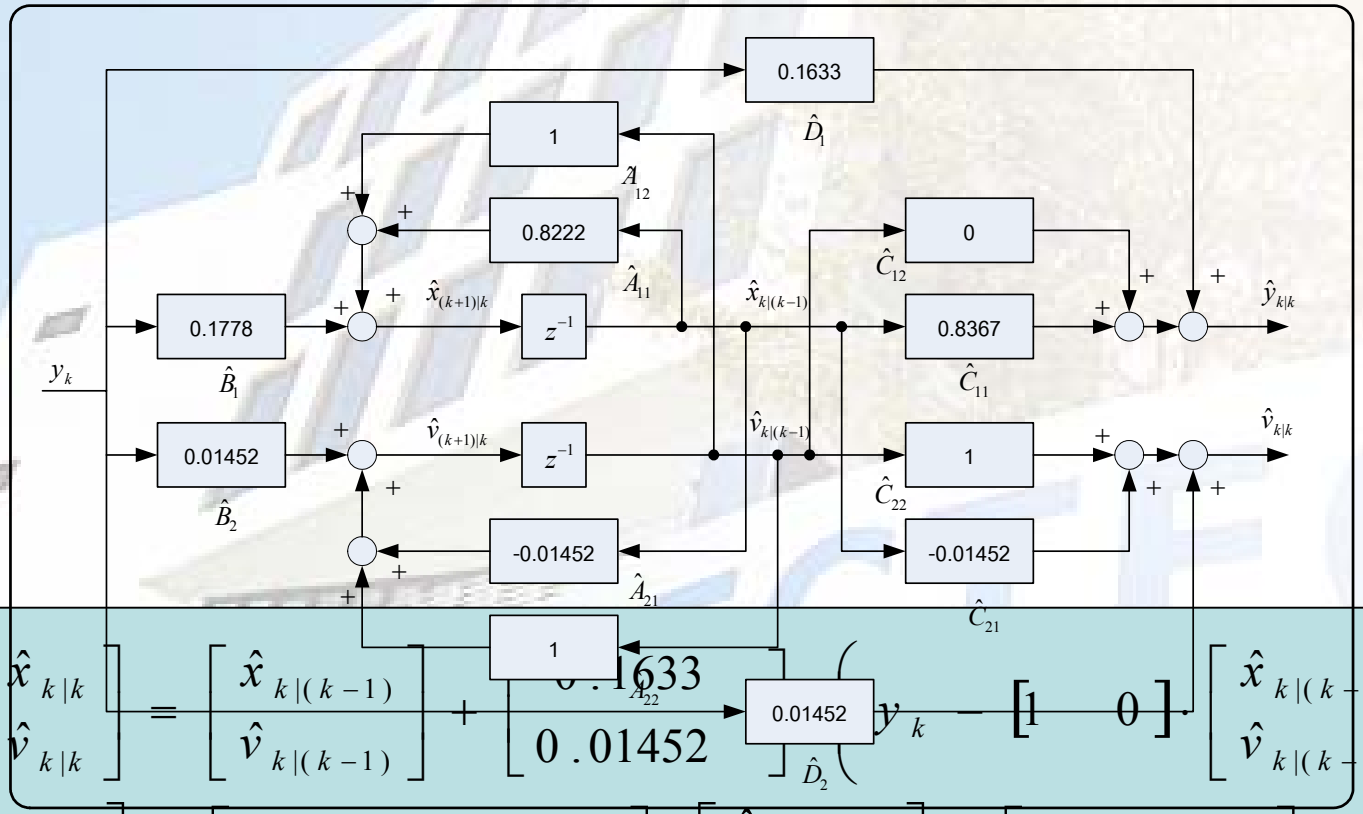
$$\begin{bmatrix} x_{k+1} \\ v_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_k \\ v_k \end{bmatrix} + \tilde{v}_k$$

$$y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_k \\ v_k \end{bmatrix} + \tilde{v}_k$$

Specification

- Estimate Position converge to Original Position
- Accurate Estimate Velocity
- Reduce Noise
- Robustness (->Fixed Point)

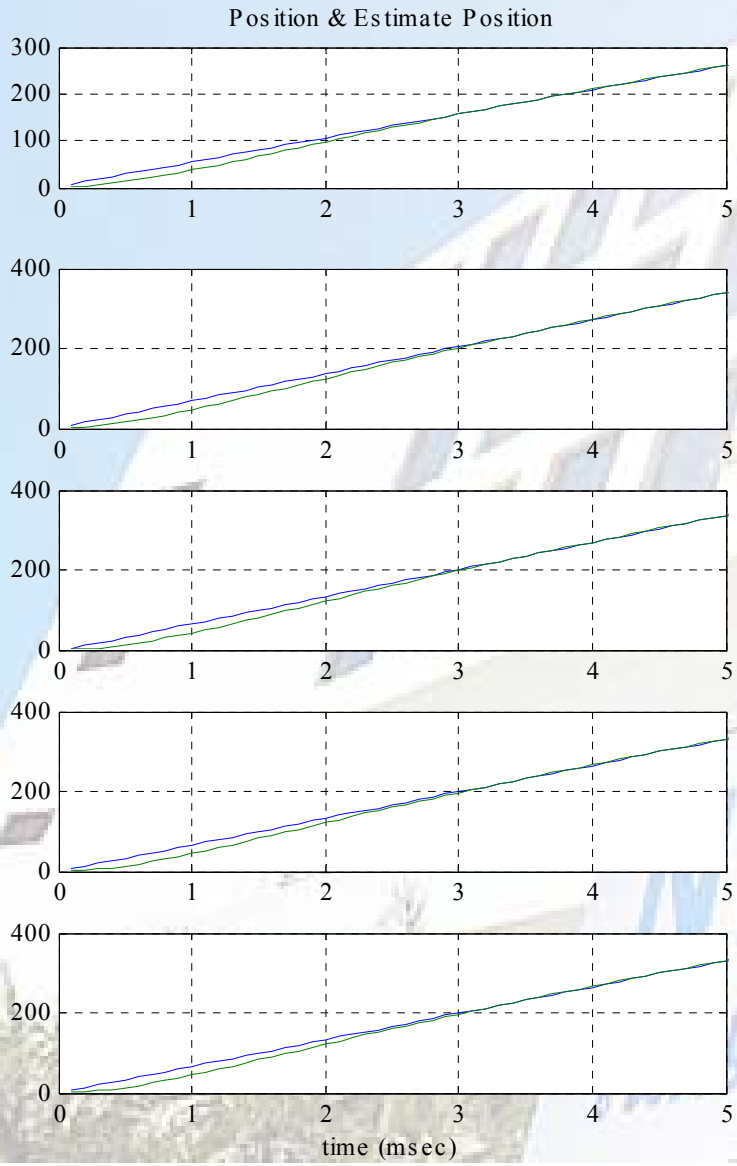
Kalman Filter

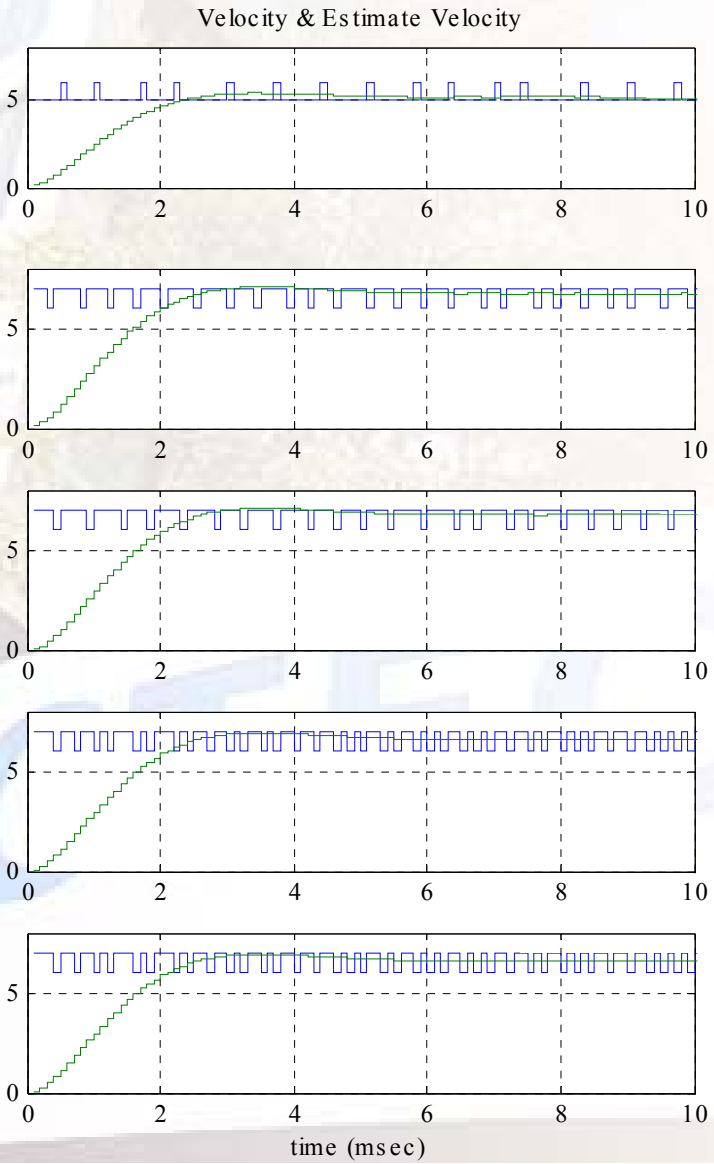


$$\begin{bmatrix} \hat{x}_{k|k} \\ \hat{v}_{k|k} \end{bmatrix} = \begin{bmatrix} \hat{x}_{k|(k-1)} \\ \hat{v}_{k|(k-1)} \end{bmatrix} + \begin{bmatrix} 0.1633 \\ 0.01452 \end{bmatrix} \left(y_k - \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \hat{x}_{k|(k-1)} \\ \hat{v}_{k|(k-1)} \end{bmatrix} \right)$$

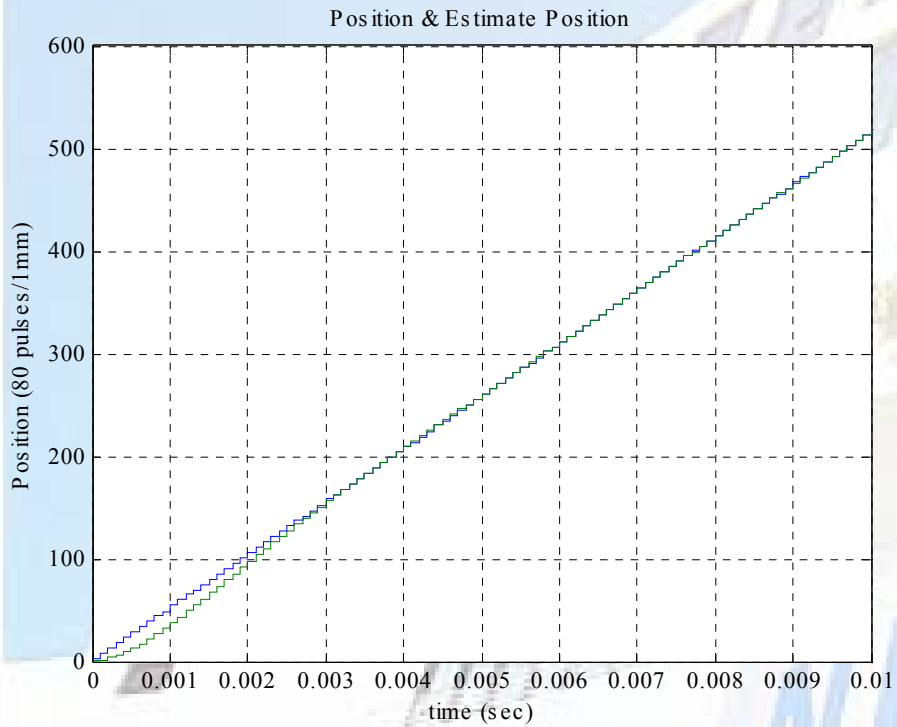
$$\begin{bmatrix} \hat{x}_{(k+1)|k} \\ \hat{v}_{(k+1)|k} \end{bmatrix} = \begin{bmatrix} 0.8222 & 1 \\ -0.01452 & 1 \end{bmatrix} \cdot \begin{bmatrix} \hat{x}_{k|(k-1)} \\ \hat{v}_{k|(k-1)} \end{bmatrix} + \begin{bmatrix} 0.1778 \\ 0.01452 \end{bmatrix} \cdot y_k$$

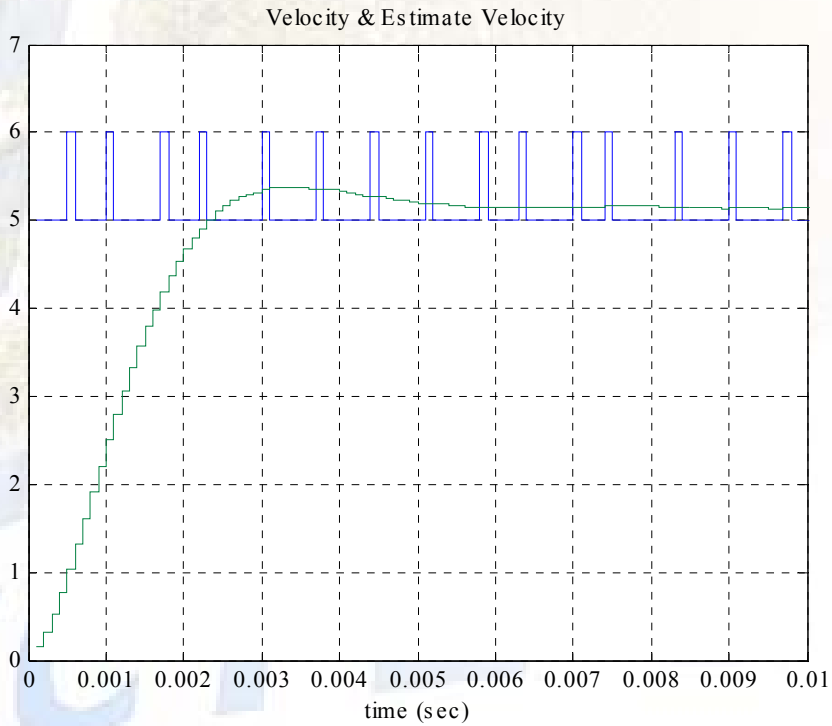
Kalman Filter Response





Kalman Filter Response (s1)

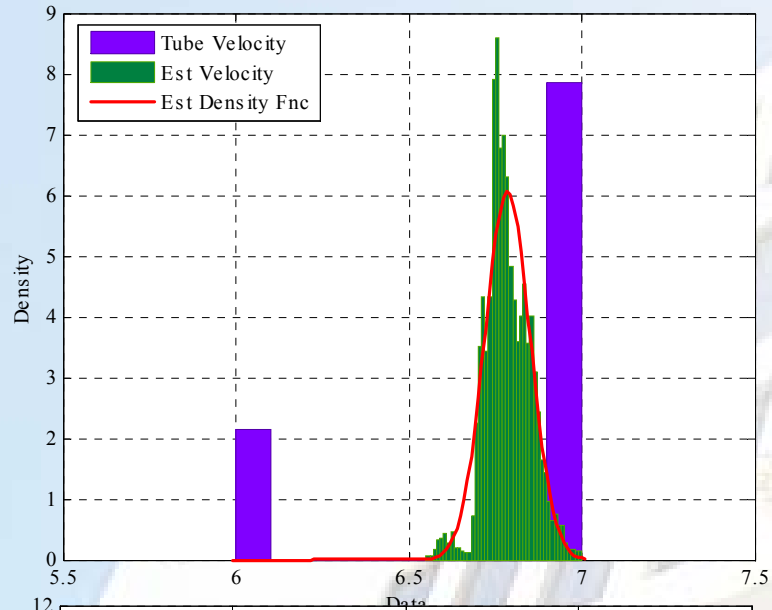


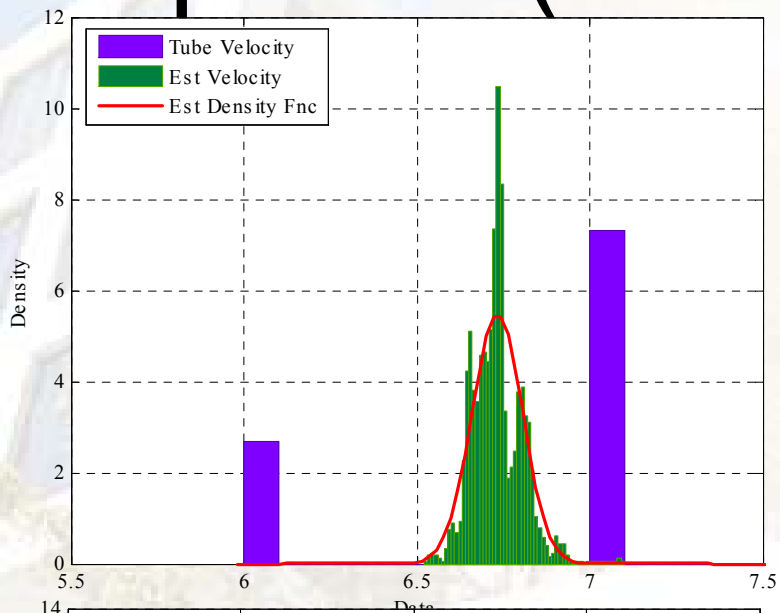


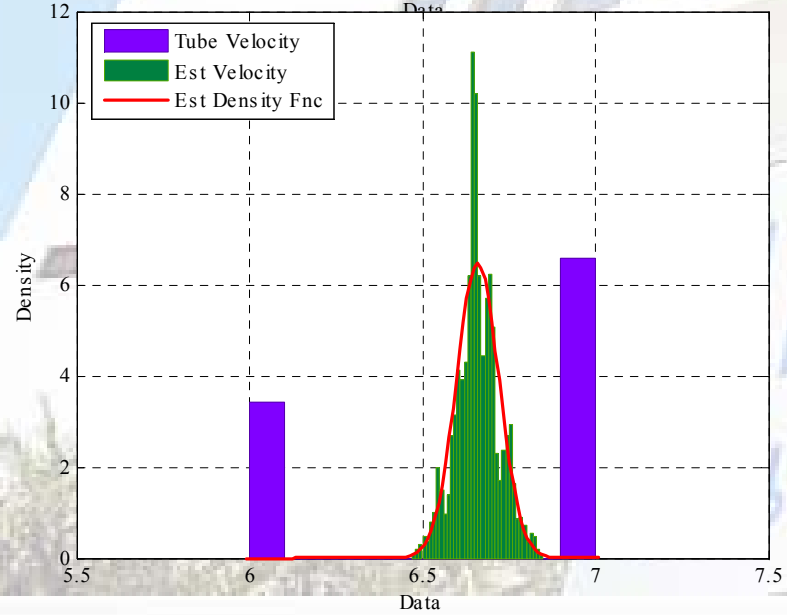
Kalman Filter Response

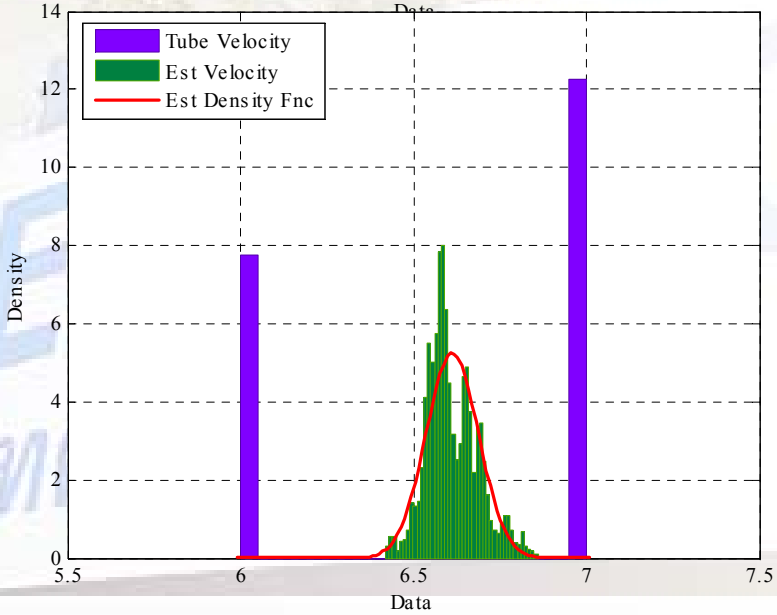
ครั้งที่	Mean $E\{(x_{k+1} - x_k)\}$	Mean $E\{\hat{v}_k\}$	Variance $E\left\{(v_k - \bar{v}_k)\cdot(v_k - \bar{v}_k)^T\right\}$	Variance $E\left\{(v_k - \bar{\hat{v}}_k)\cdot(v_k - \bar{\hat{v}}_k)^T\right\}$
1	**5.2361**	**5.23717**	**0.181621**	**0.0184262**
2	**6.7736**	**6.77375**	**0.175178**	**0.0066362**
3	**6.7728**	**6.77264**	**0.176415**	**0.0069034**
4	**6.6392**	**6.63918**	**0.230669**	**0.0087330**
5	**6.6276**	**6.62762**	**0.233765**	**0.0120377**

Kalman Filter Response (s2-4)









Implement on Fixed-Point DSP

$$\begin{bmatrix} \hat{x}_{(k+1)|k} \\ \hat{v}_{(k+1)|k} \end{bmatrix} = \begin{bmatrix} 0.8222 & 1 \\ -0.01452 & 1 \end{bmatrix} \cdot \begin{bmatrix} \hat{x}_{k|(k-1)} \\ \hat{v}_{k|(k-1)} \end{bmatrix} + \begin{bmatrix} 0.1778 \\ 0.01452 \end{bmatrix} \cdot y_k$$

(1/1024) = 0.0009765625

$$\begin{bmatrix} \hat{x}_{(k+1)|k} \\ \hat{v}_{(k+1)|k} \end{bmatrix} = \begin{bmatrix} 0.8222656250 & 1 \\ -0.0146484375 & 1 \end{bmatrix} \cdot \begin{bmatrix} \hat{x}_{k|(k-1)} \\ \hat{v}_{k|(k-1)} \end{bmatrix} + \begin{bmatrix} 0.1777343750 \\ 0.0146484375 \end{bmatrix} \cdot y_k$$

Round-Off Error

1. Parameter Round-Off Error

2. Variable Round-Off Error

Parameter Round-Off Error

<Offline Analysis>

Change Characteristic

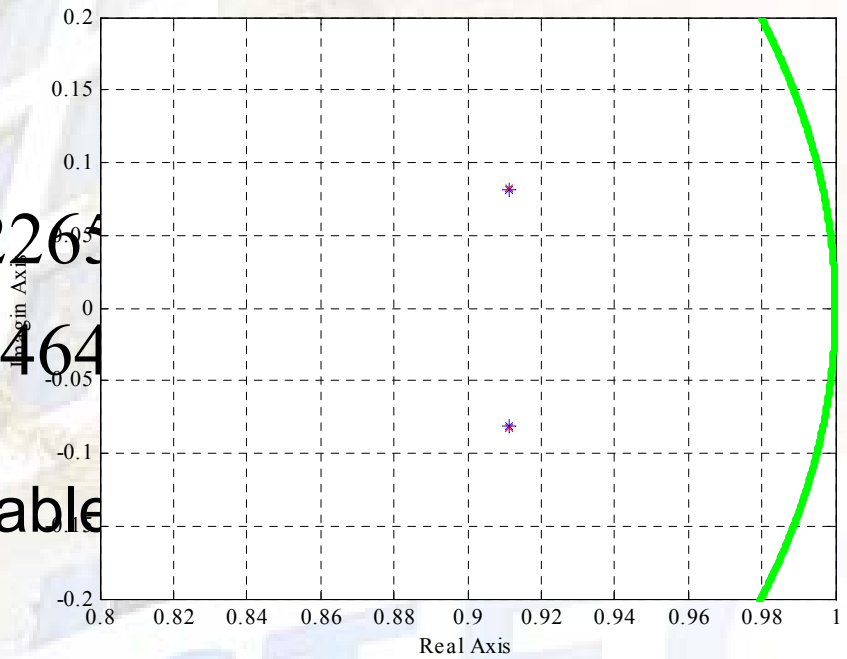
1. Roots Perturbation => Stability

2. Convergence

Check Stability

$$\hat{A} = \begin{bmatrix} 0.82226 \\ -0.01464 \end{bmatrix}$$

Stable



Eigen Value = $0.9111328125 \pm 0.0821648372 \cdot i$

Stay in Unit Circle. => Stable

Check Convergence

$$A = A + \hat{B} \cdot C+$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.8222656250 & 1 \\ -0.0146484375 & 1 \end{bmatrix} \cdot C + \begin{bmatrix} 0.1777343750 \\ ? \\ 0.0146484375 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (0.8222656250 + 0.1777343750) & 1 \\ (-0.0146484375 + 0.0146484375) & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

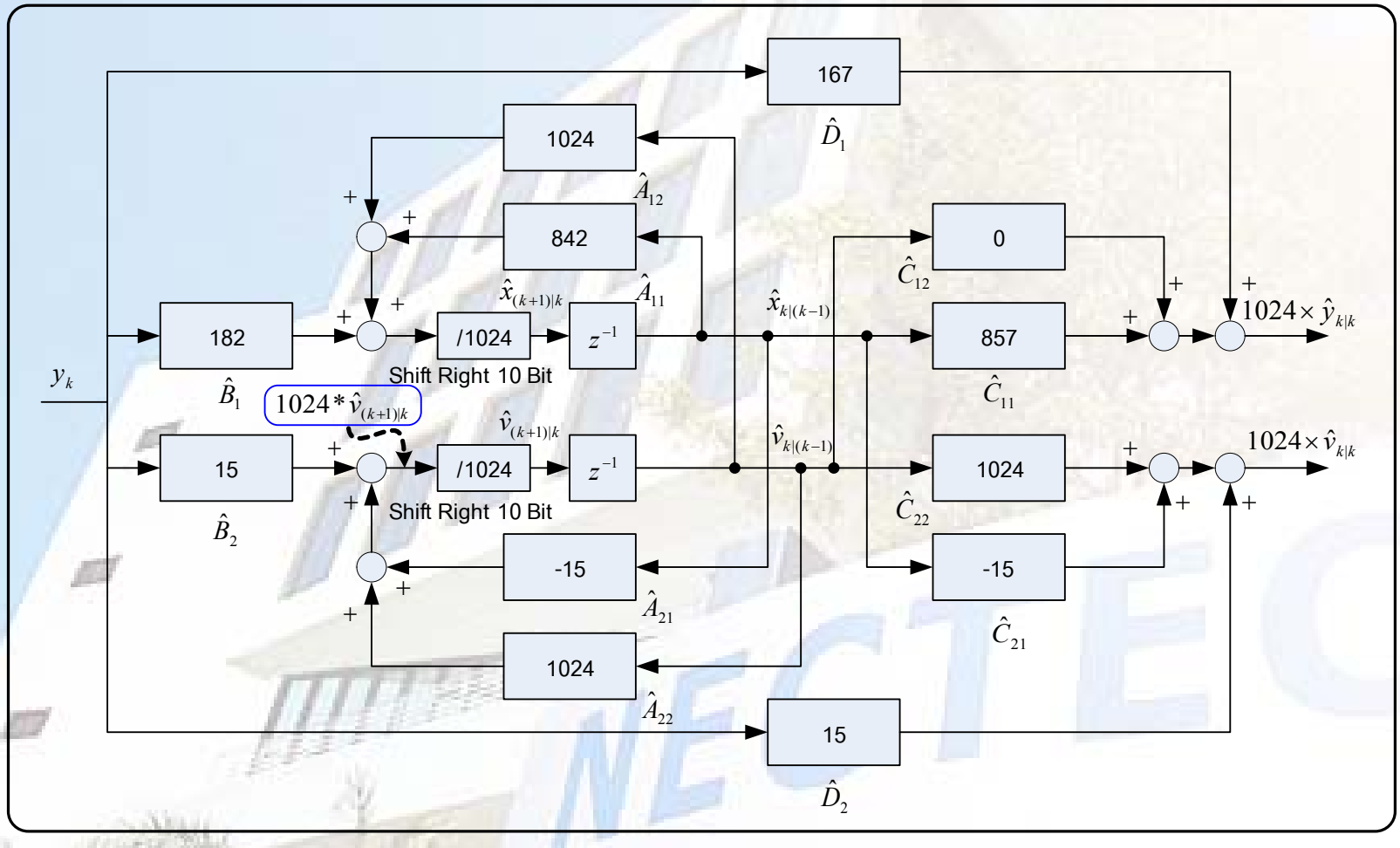
Converge

Variable Round-Off Error

<Online Calculate>

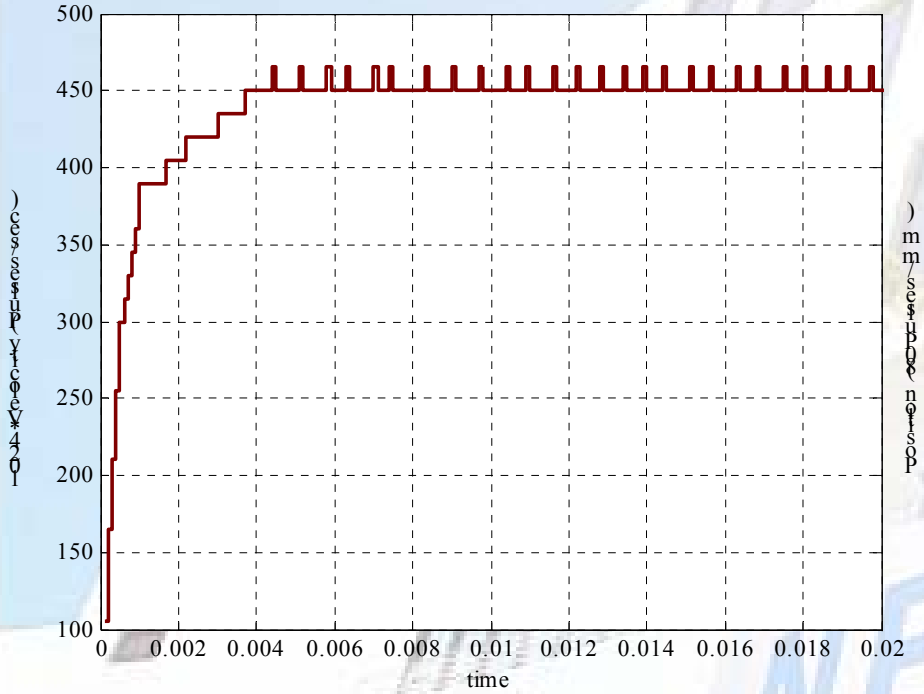
Round off in Algebraic Calculation

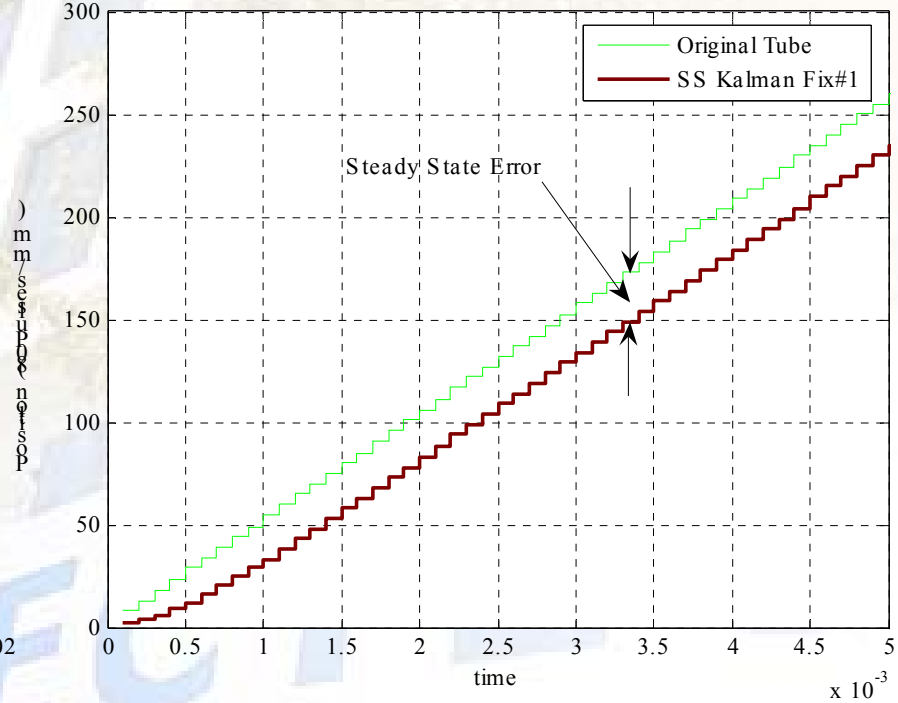
Example 1



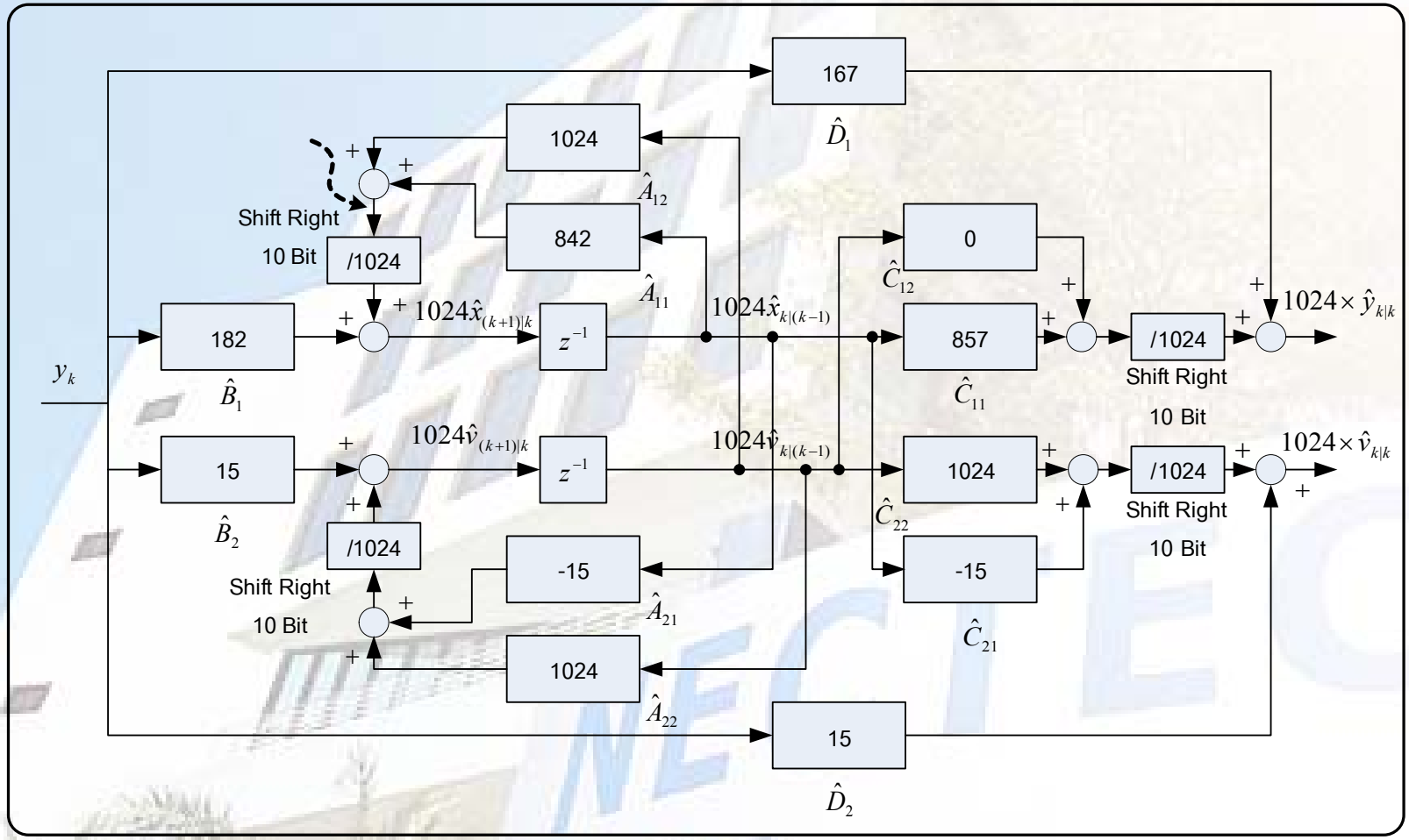
$$\left\| \text{Round} - \text{Off Error} \right\|_{\infty} \leq 1$$

Bad Result


500
450
400
350
300
250
200
150
100
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
0.02
time

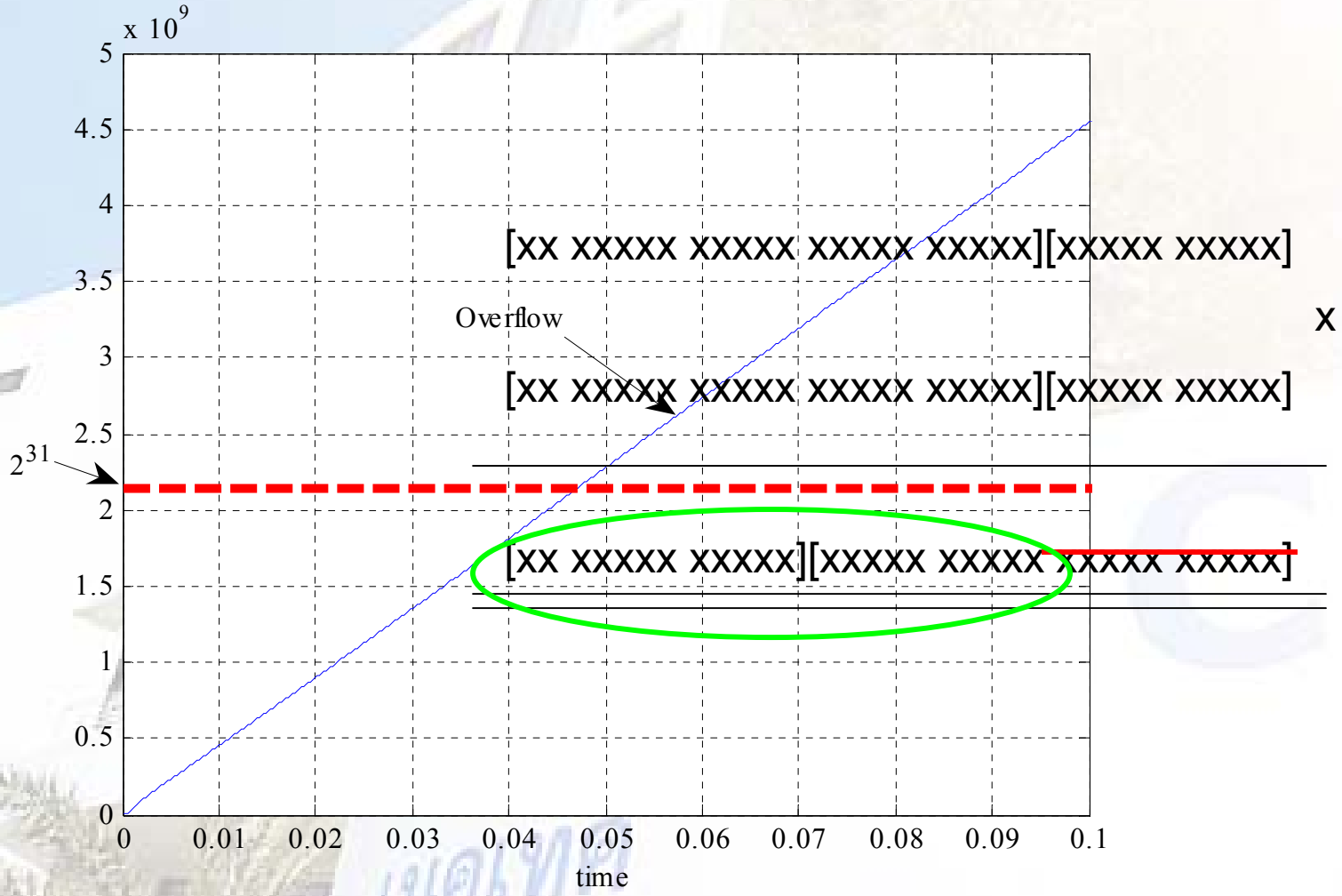

Original Tube
SS Kalman Fix#1
Steady State Error
300
250
200
150
100
50
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
time
x 10^-3

Example 2



$$\left\| \text{Round} - \text{Off Error} \right\|_{\infty} \leq (1/1024)$$

Overflow Problem



Solution

$$(a \cdot x)/1024 =$$

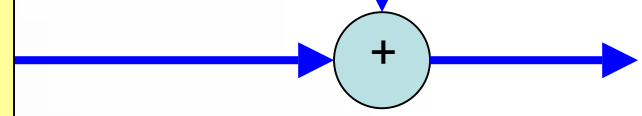
$$x = (m \cdot 1024) + n$$

[xx xxxxx xxxxx xxxxx xxxxx][00000 00000]
[xx xxxxx xxxxx xxxxx xxxxx][xxxxx xxxxx] x

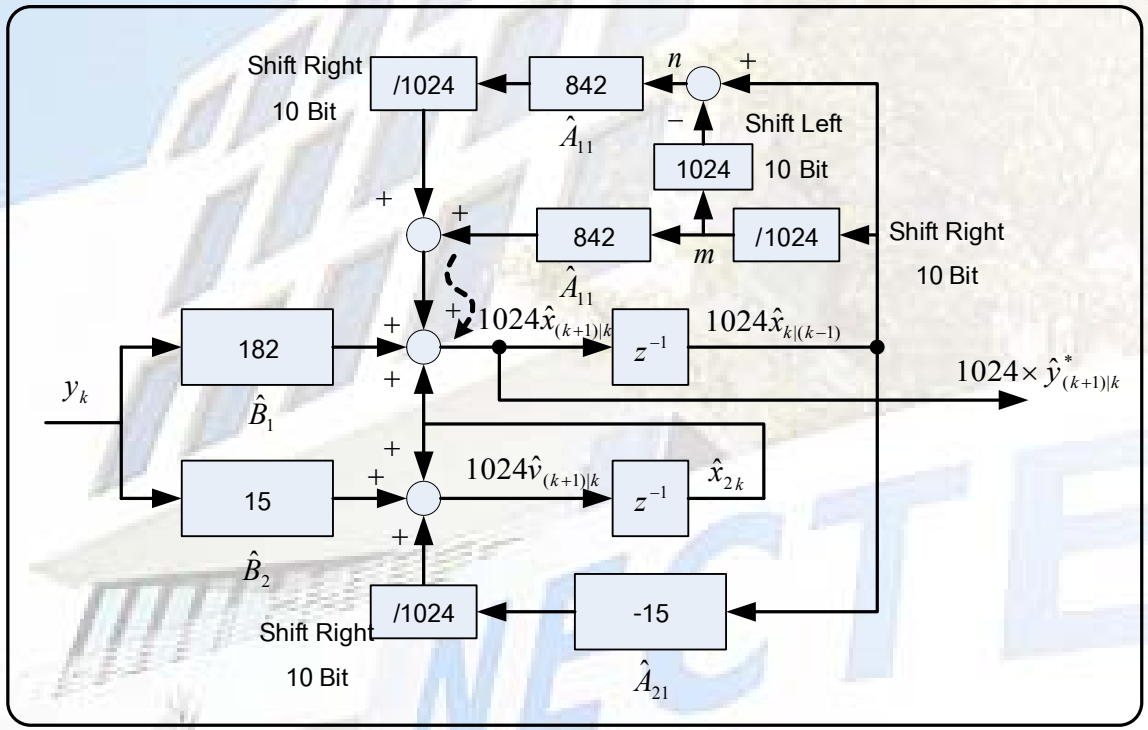
[xx xxxxx xxxxx xxxxx xxxxx][xxxxx xxxxx]

[00 00000 00000 00000 00000] [xxxxx xxxxx]
[xx xxxxx xxxxx xxxxx xxxxx][xxxxx xxxxx] x

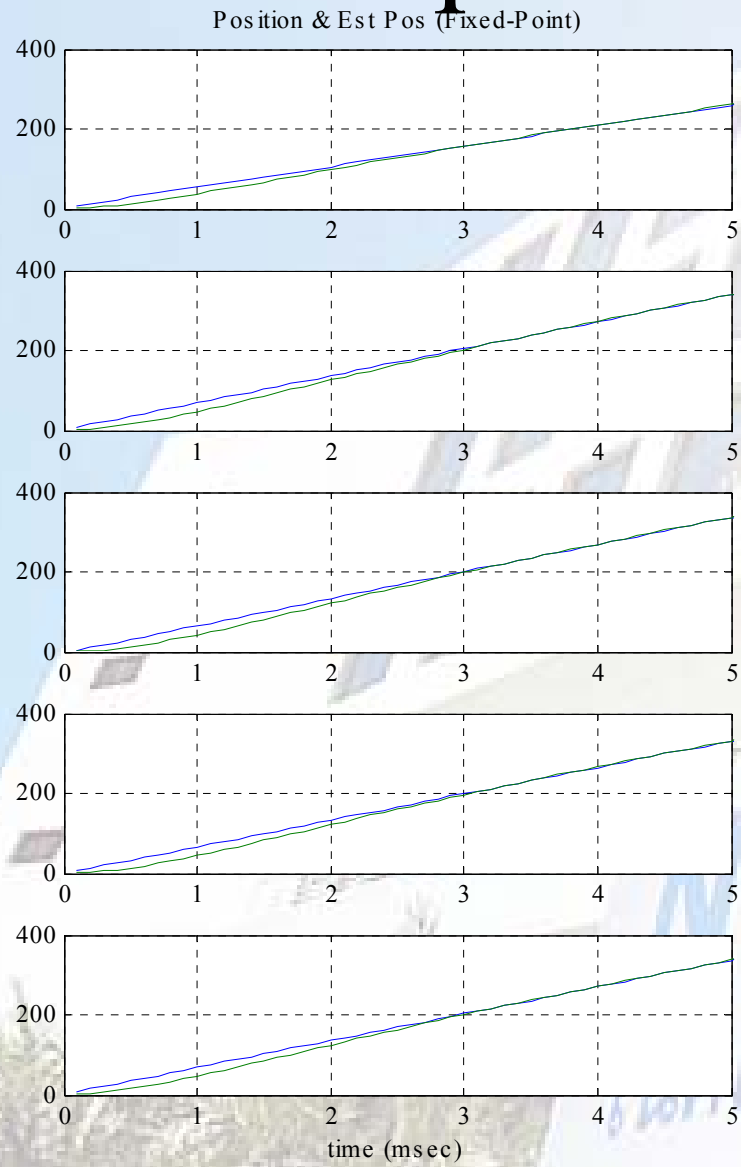
[xx xxxxx xxxxx][xxxxx xxxxx xxxxx xxxxx]

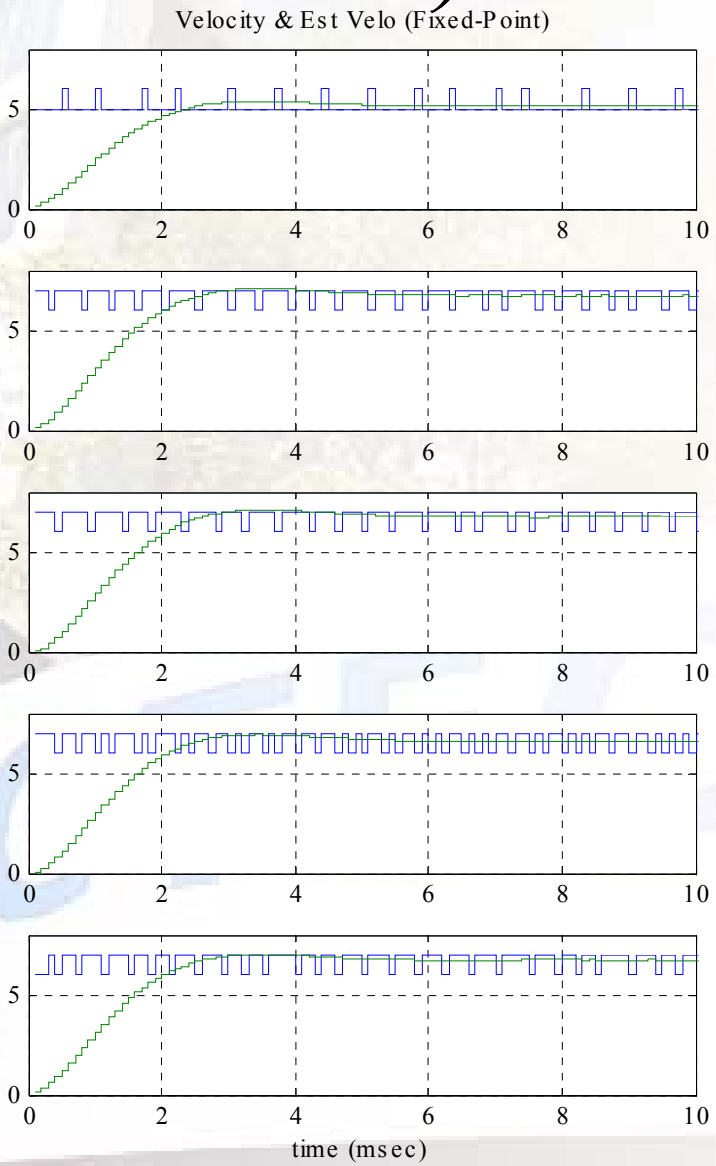


Example 2 (Modified)

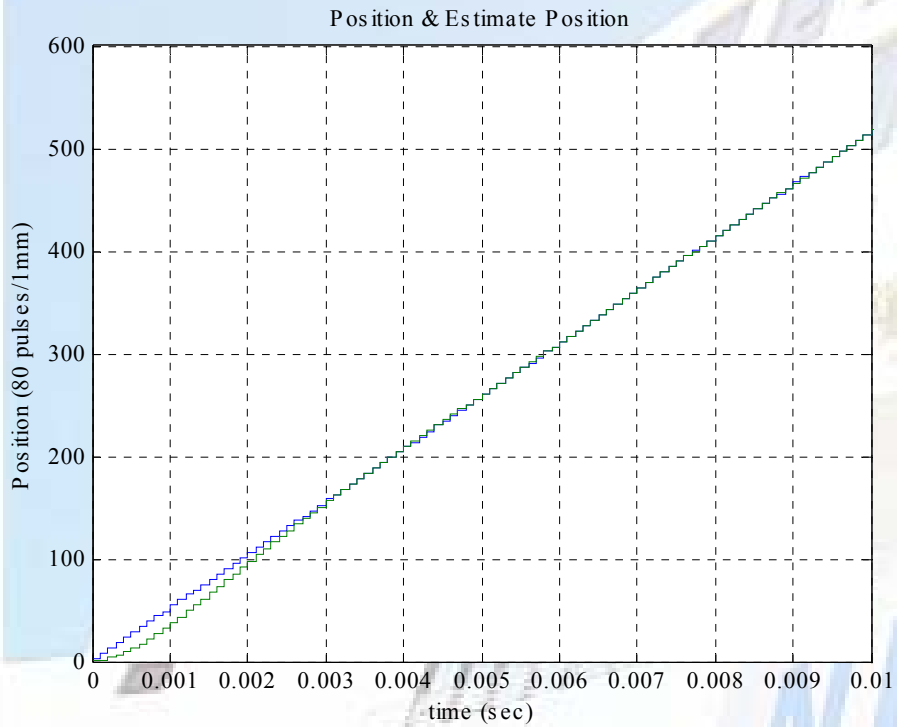


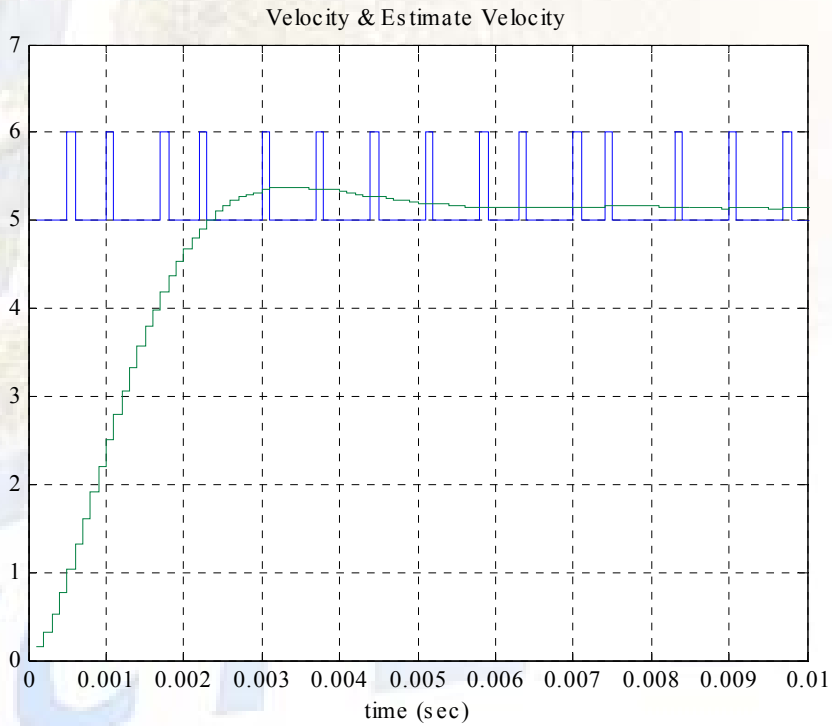
Response (Pos & Vel)



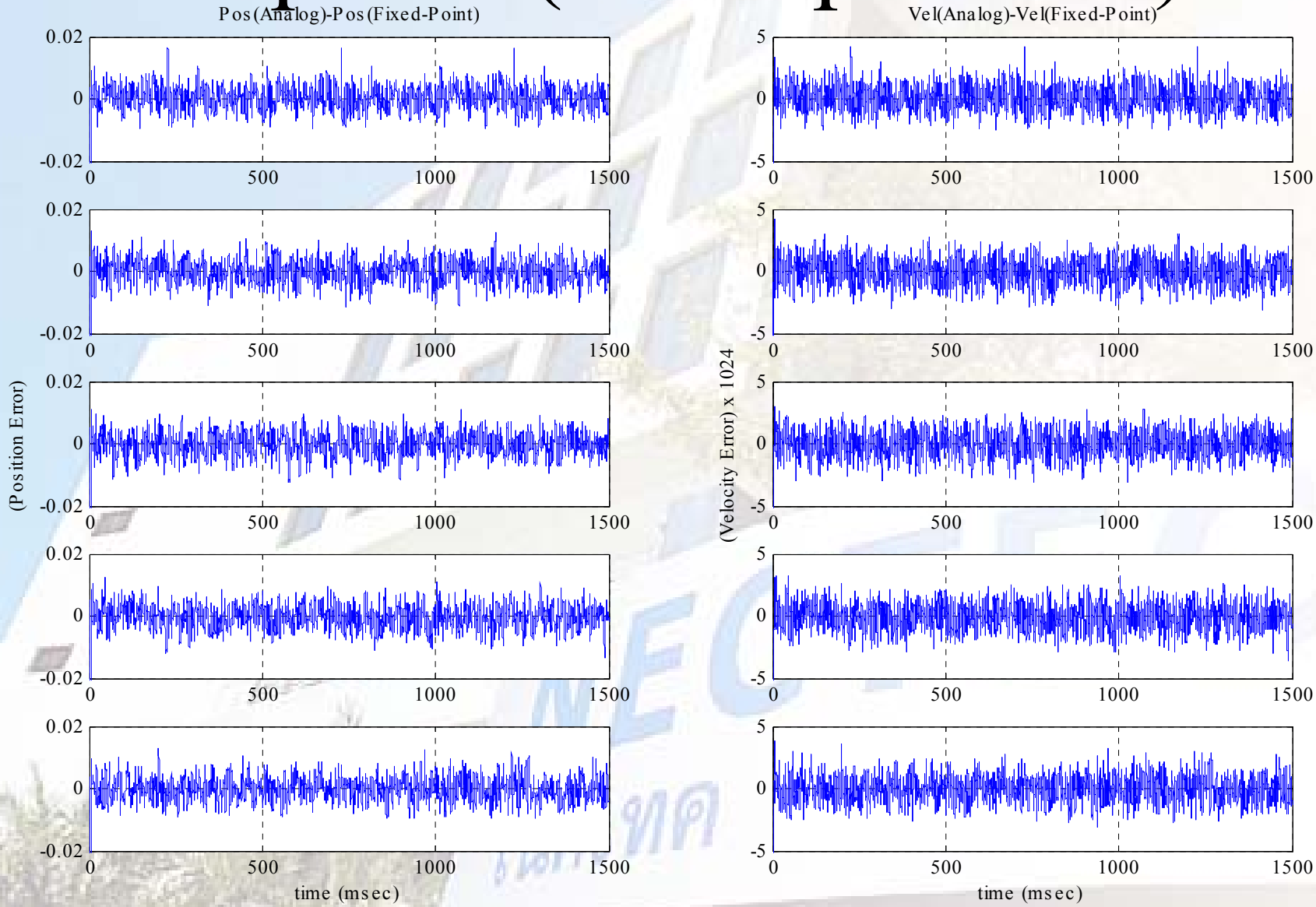


Response (Pos & Vel s1.)

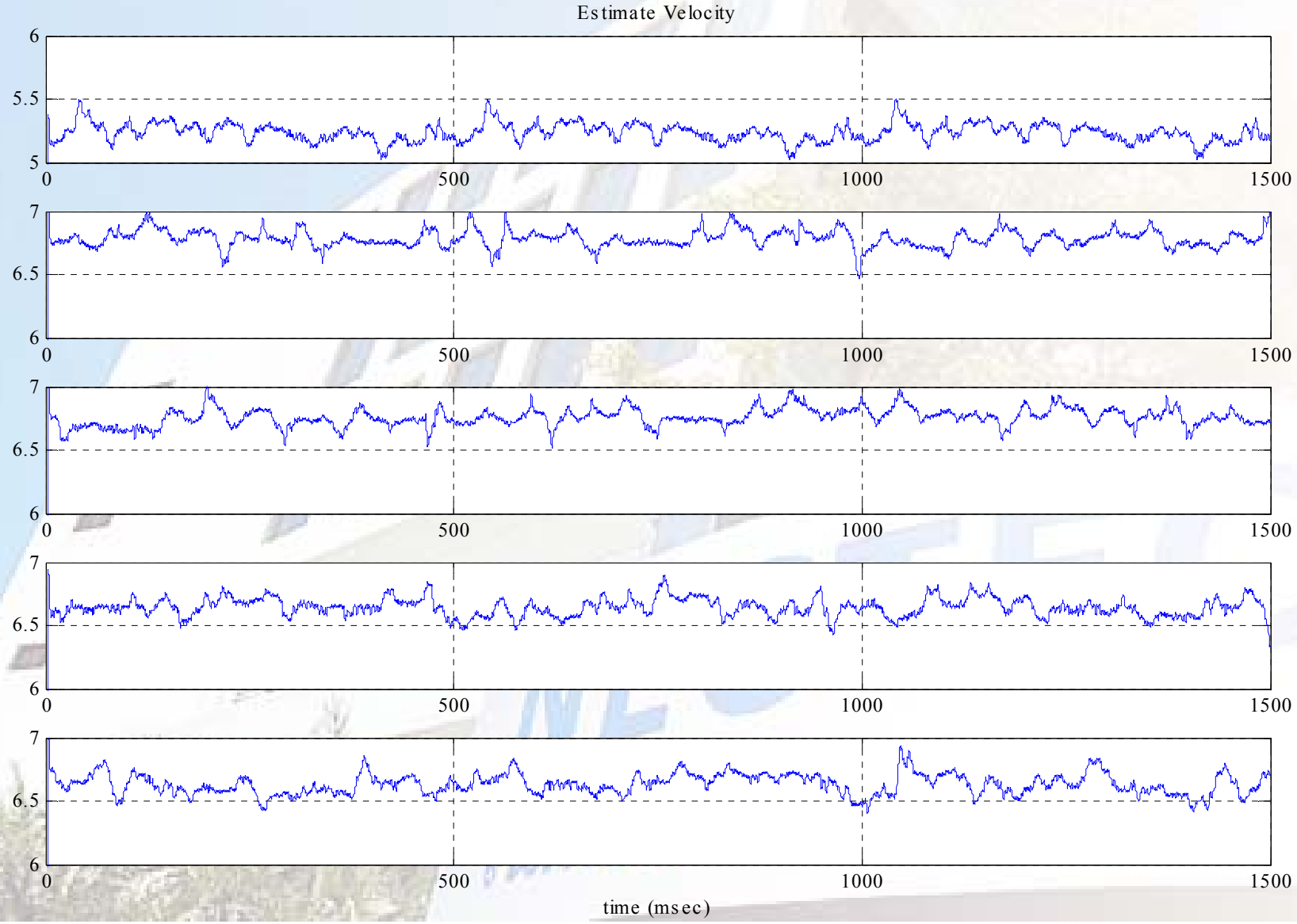




Response (Fixed point err)



Response (Estimate Velocity)



Conclusion

1. Parameter Round-Off Error
   -> Stability
   -> Convergence

2. Variable Round-Off Error
   -> Integrated of round-off error

Thank you