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Five Algorithms to Optimize and Correct the Tool Path of the Five-Axis Milling Machine

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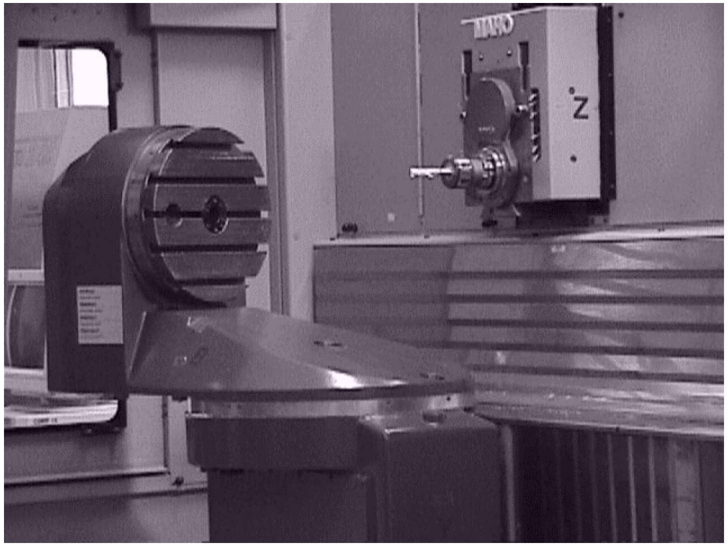
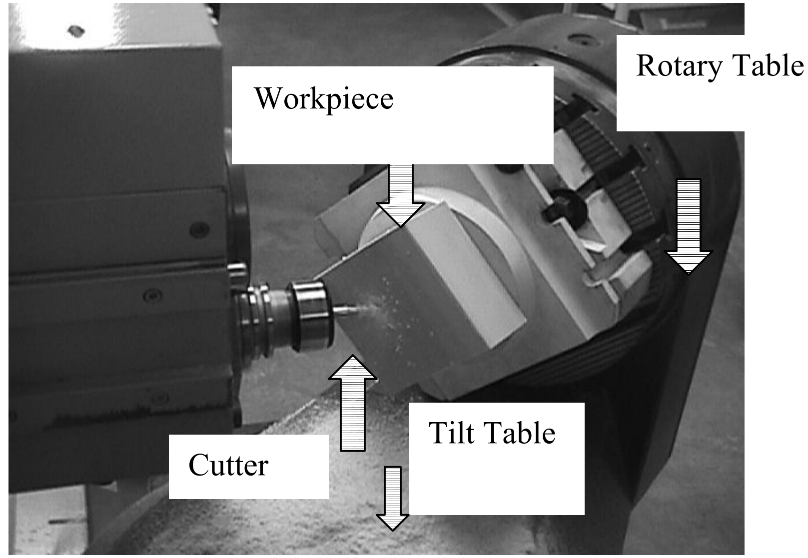
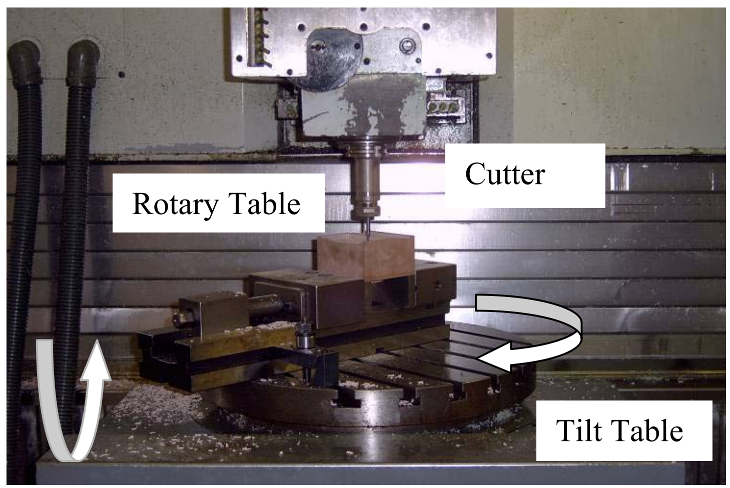
Two Royal Golden Jubilee Scholarships of the Thailand Research Fund are available for Thai Master of Ph.D. students

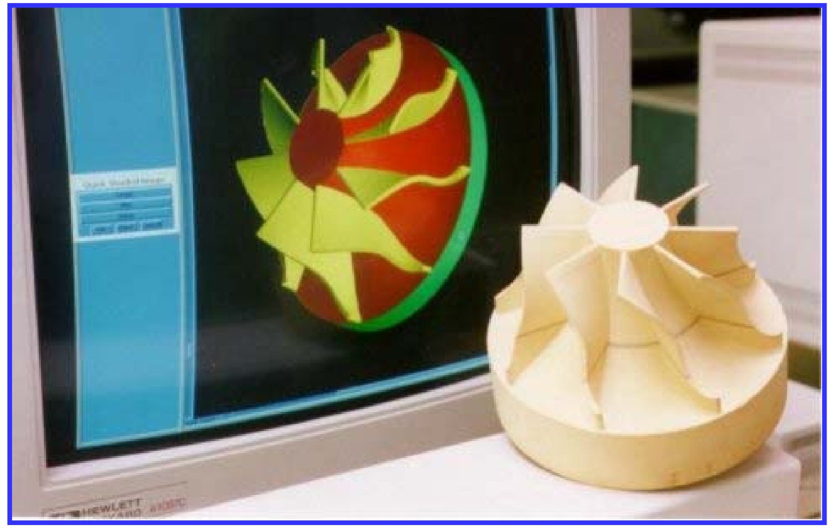
Milling Machines Maho600E and HERMLE

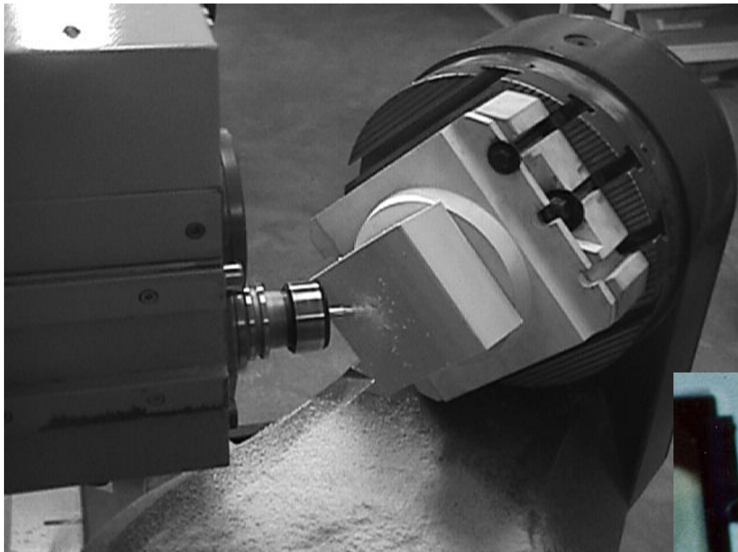


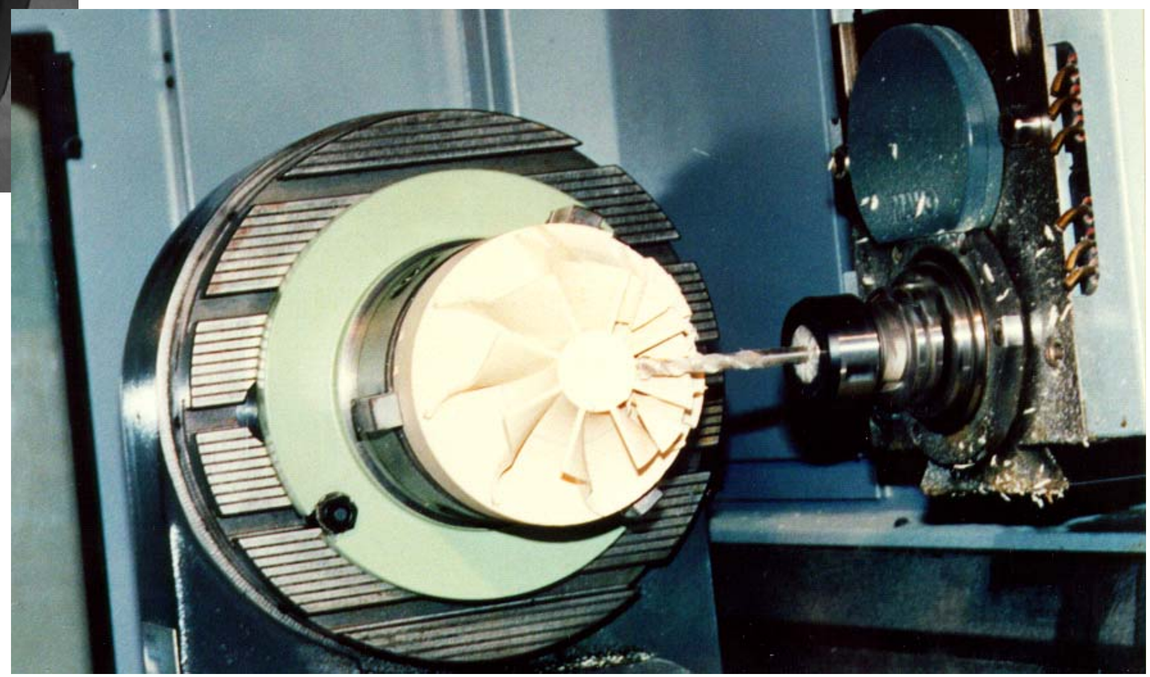




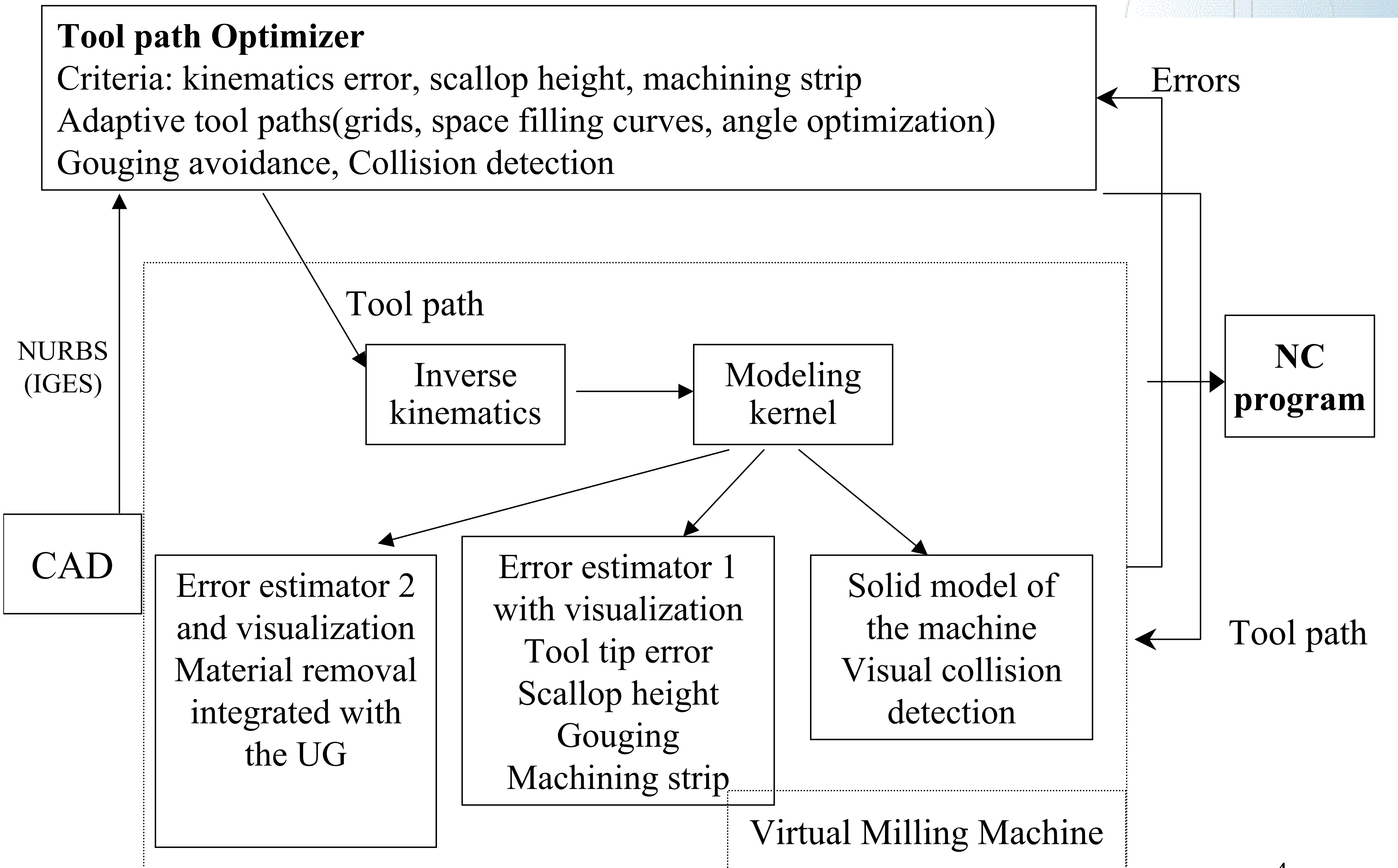
5-Axis Machining



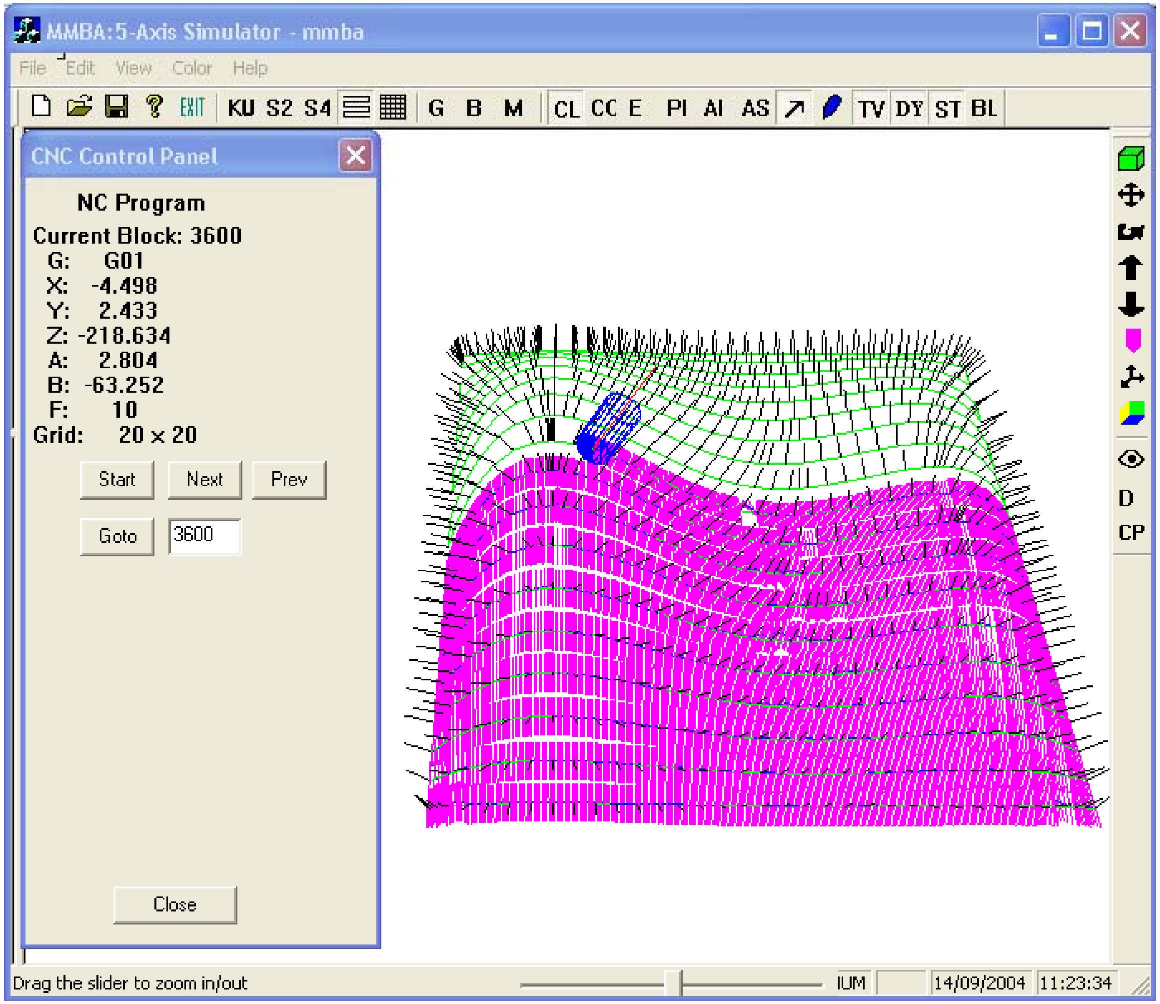




General Objective: Tool path Optimization. A Software Prototype

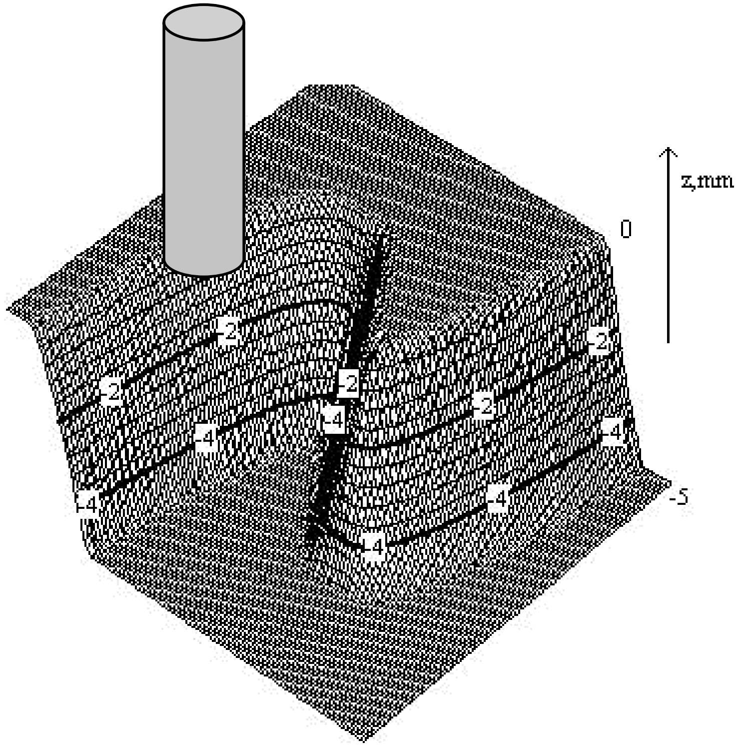


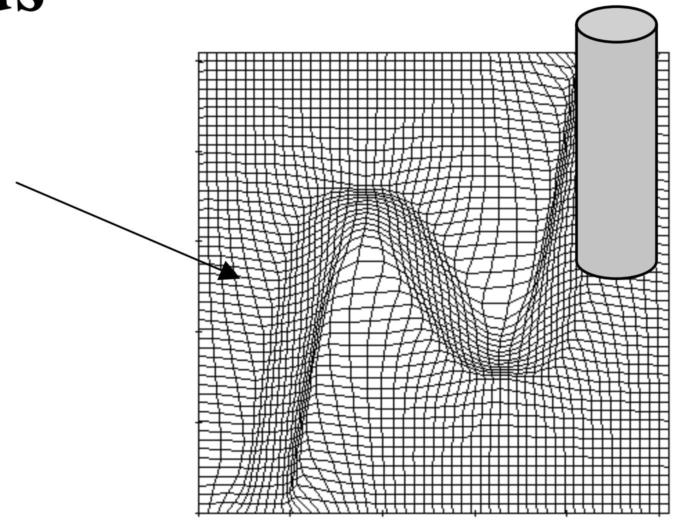
Optimization Problem

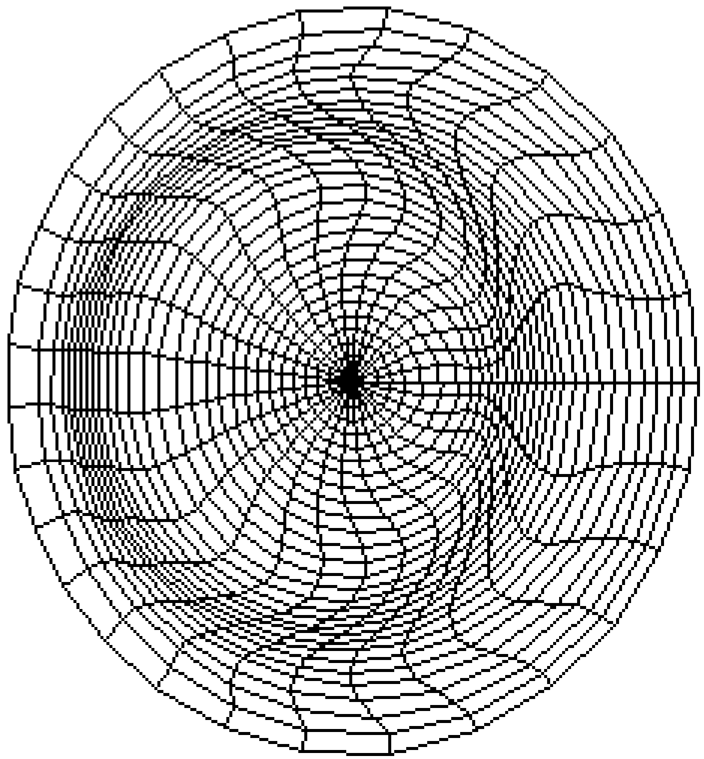
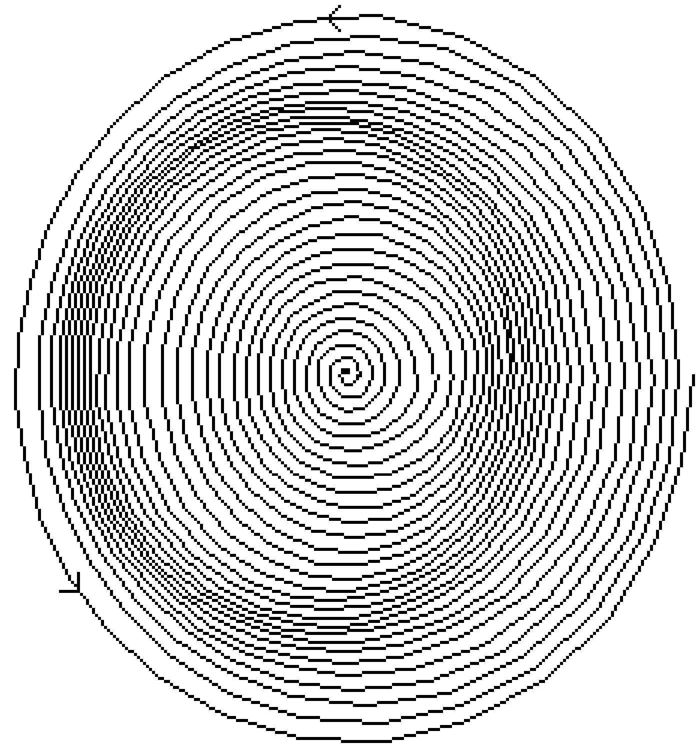


Given a surface find
a set of tool positions
and orientations
such that the surface is
cut with max accuracy
for minimum time

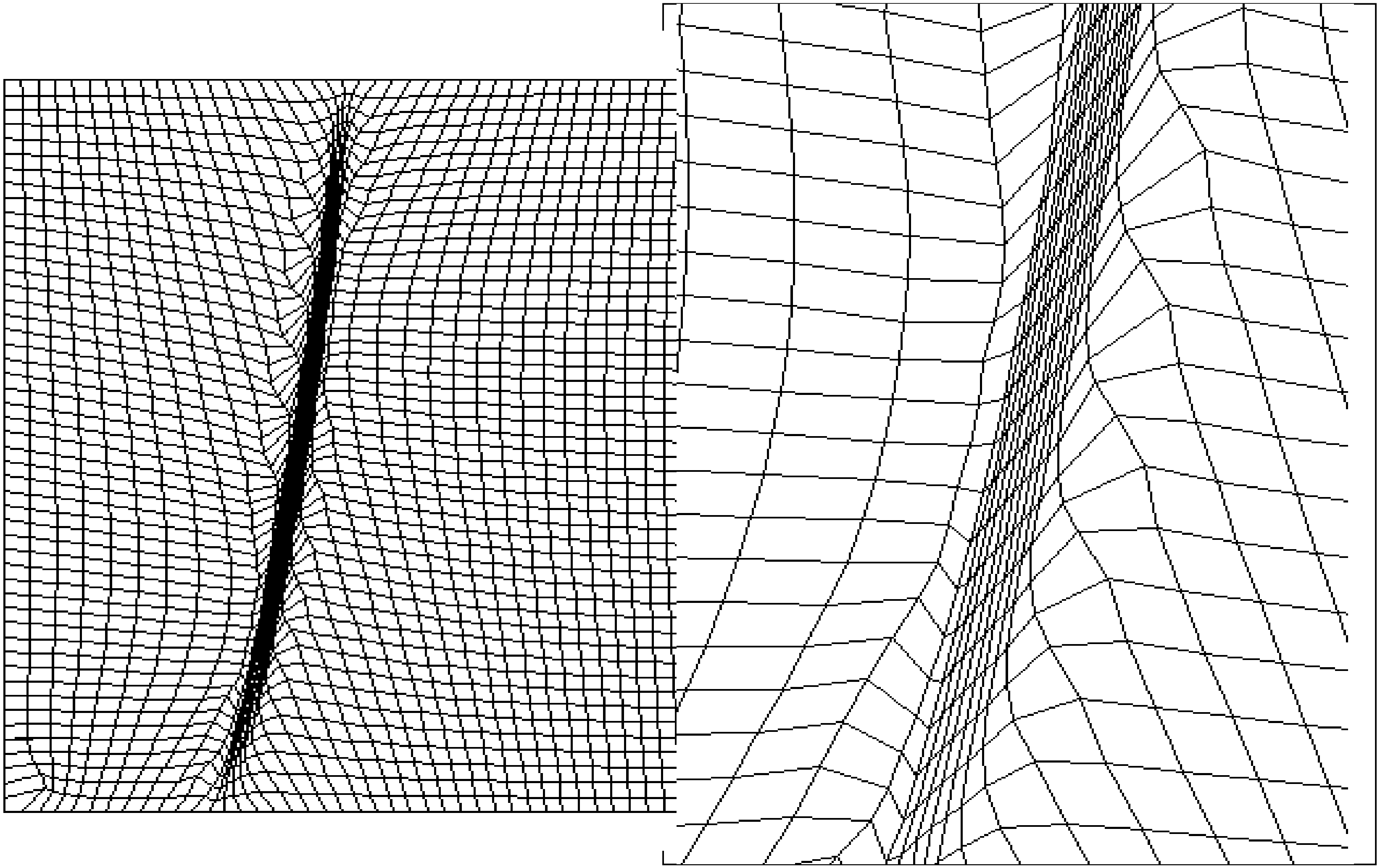
Idea 1. Curvilinear grids



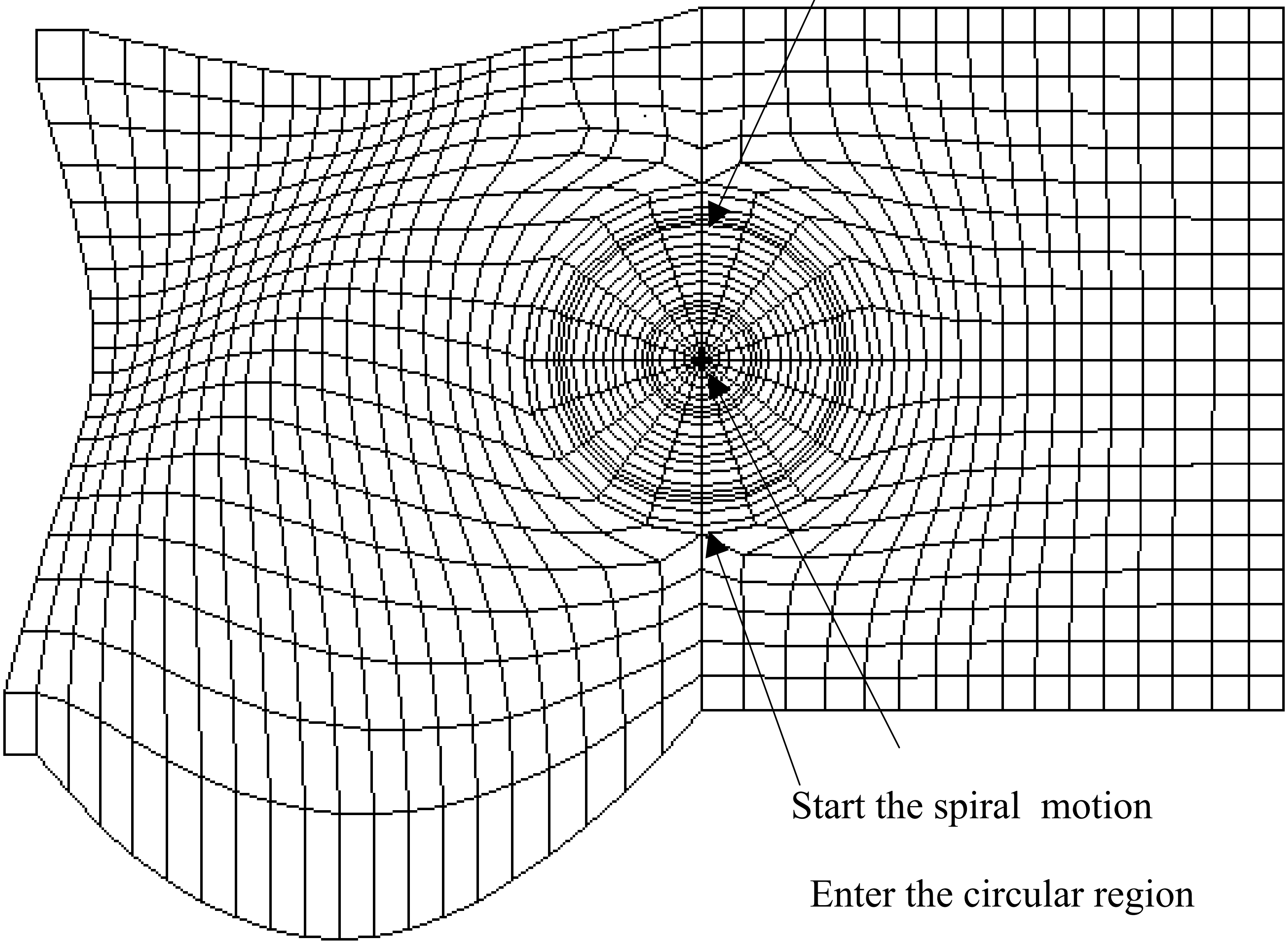


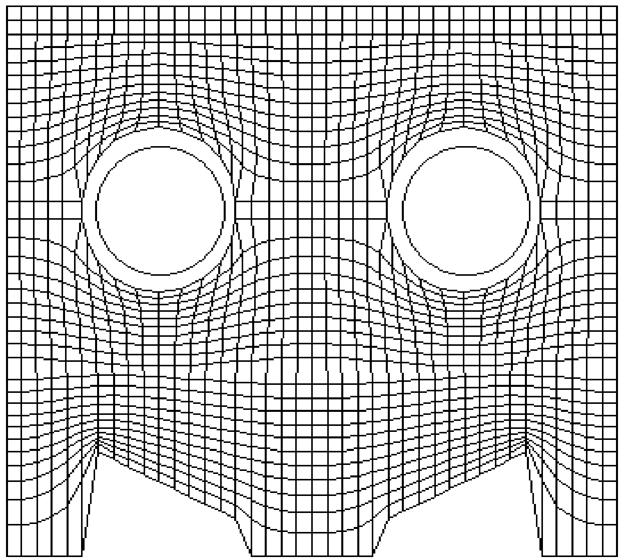
A very high density of the CL- points



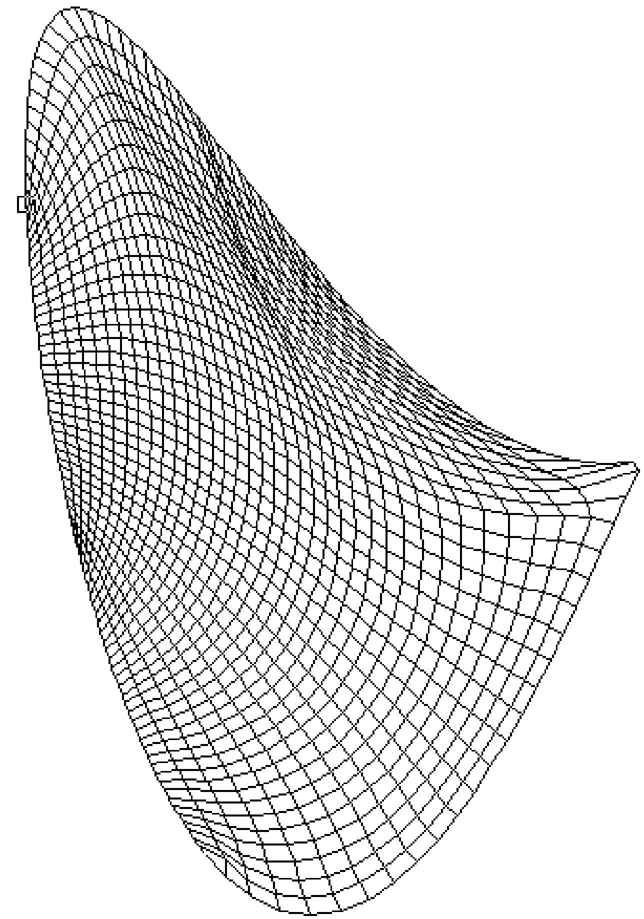
Composite patterns



Adaptation to the boundary & pockets

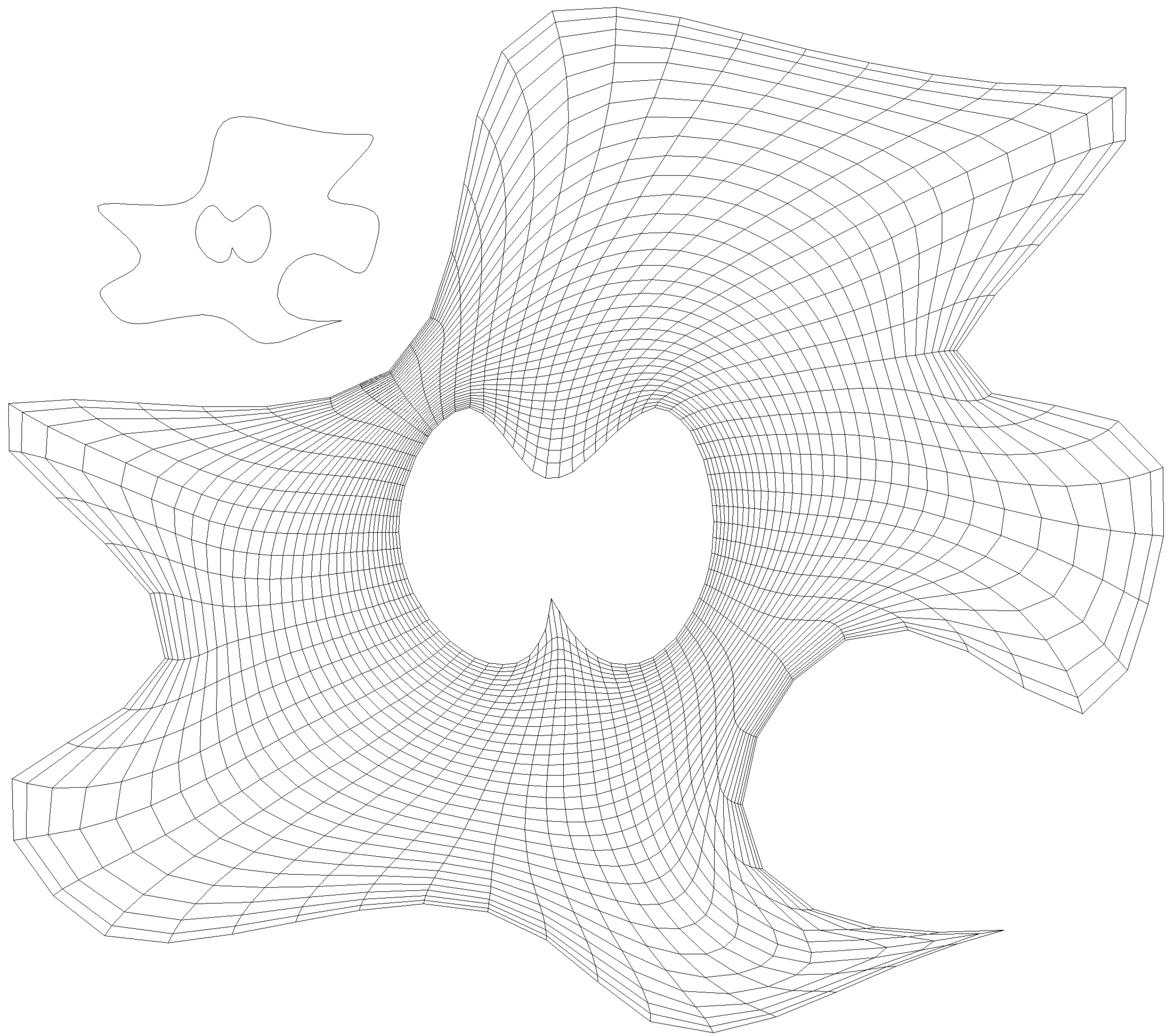


Robot with sad face

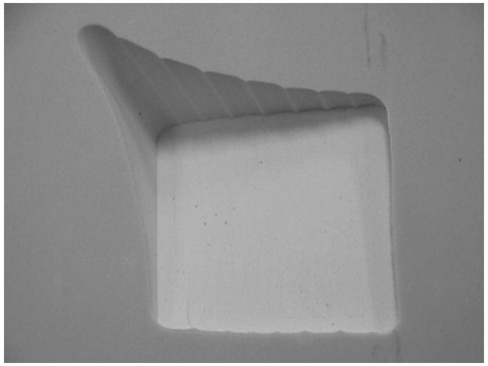


Oister

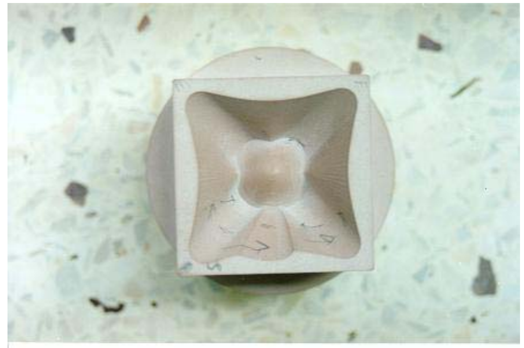
A complicated boundary & a pocket



Workpeices produced by means of adaptive tool path

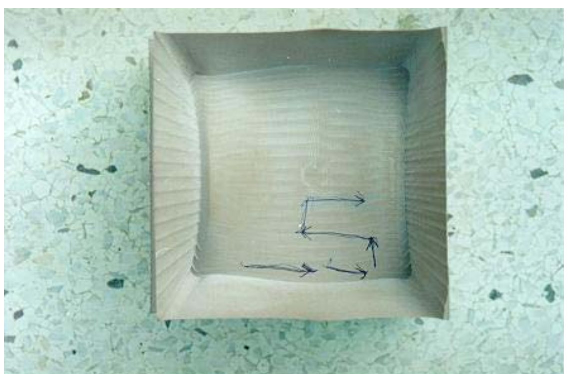


Concave-convex surface, plastic

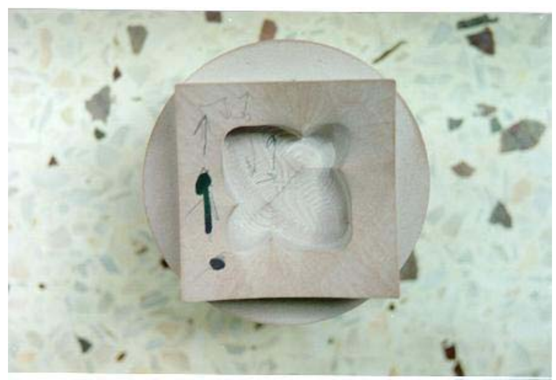


Parabolic surface, wood

Concave-convex Bezier surface, steel



Concave-convex surface, wood



Complex boundary, wood

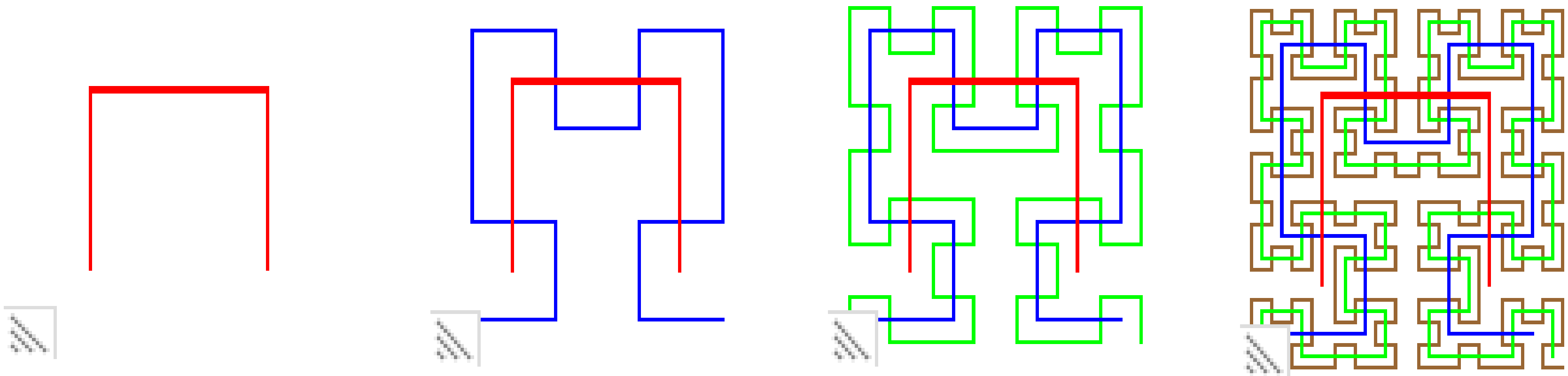
Internal boundary, wood.

Accuracy and roughness of the machined surfaces

CL points	Av. step, mm	Error decrease, % / mm	Roughness (conventional) µm	Roughness (adaptive) µm
100	3.60	34 / 0.2600	*	*
400	1.80	41 / 0.0930	*	*
900	1.20	34 / 0.0580	34.8	17.3
1600	0.90	36 / 0.0410	14.3	6.6
3600	0.60	32 / 0.0260	5.9	4.3
6400	0.45	40 / 0.0180	2.6	2.1

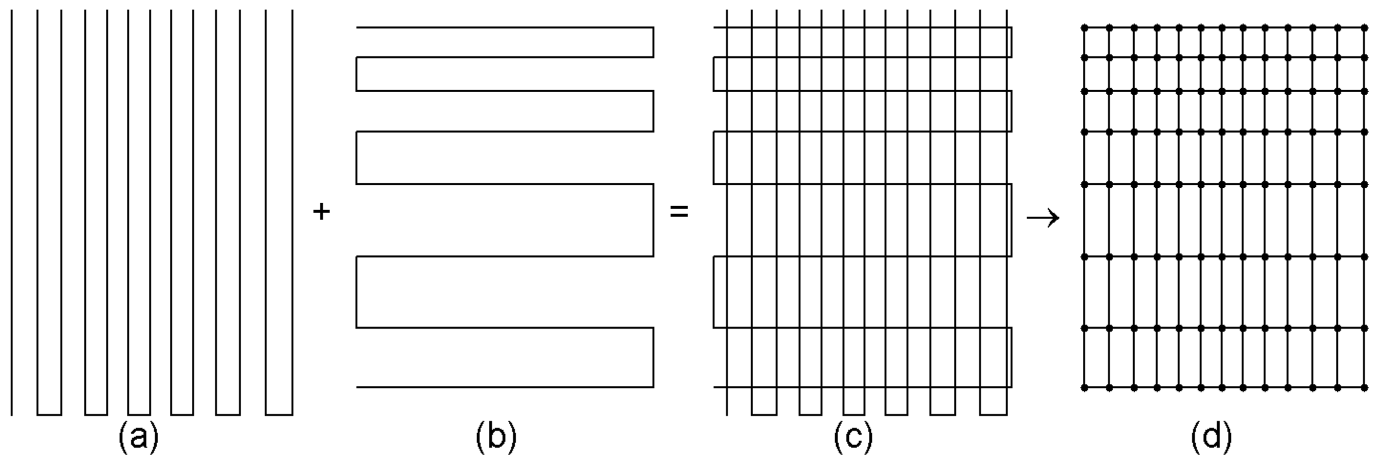
Idea 2. Space-Filling Curve

- SFC is a continuous mapping of a unit line segment onto the unit square.



Adaptive Space-Filling Curve Construction

- Overlay two iso-parametric tool paths; one in the v-direction and one in the u-direction.

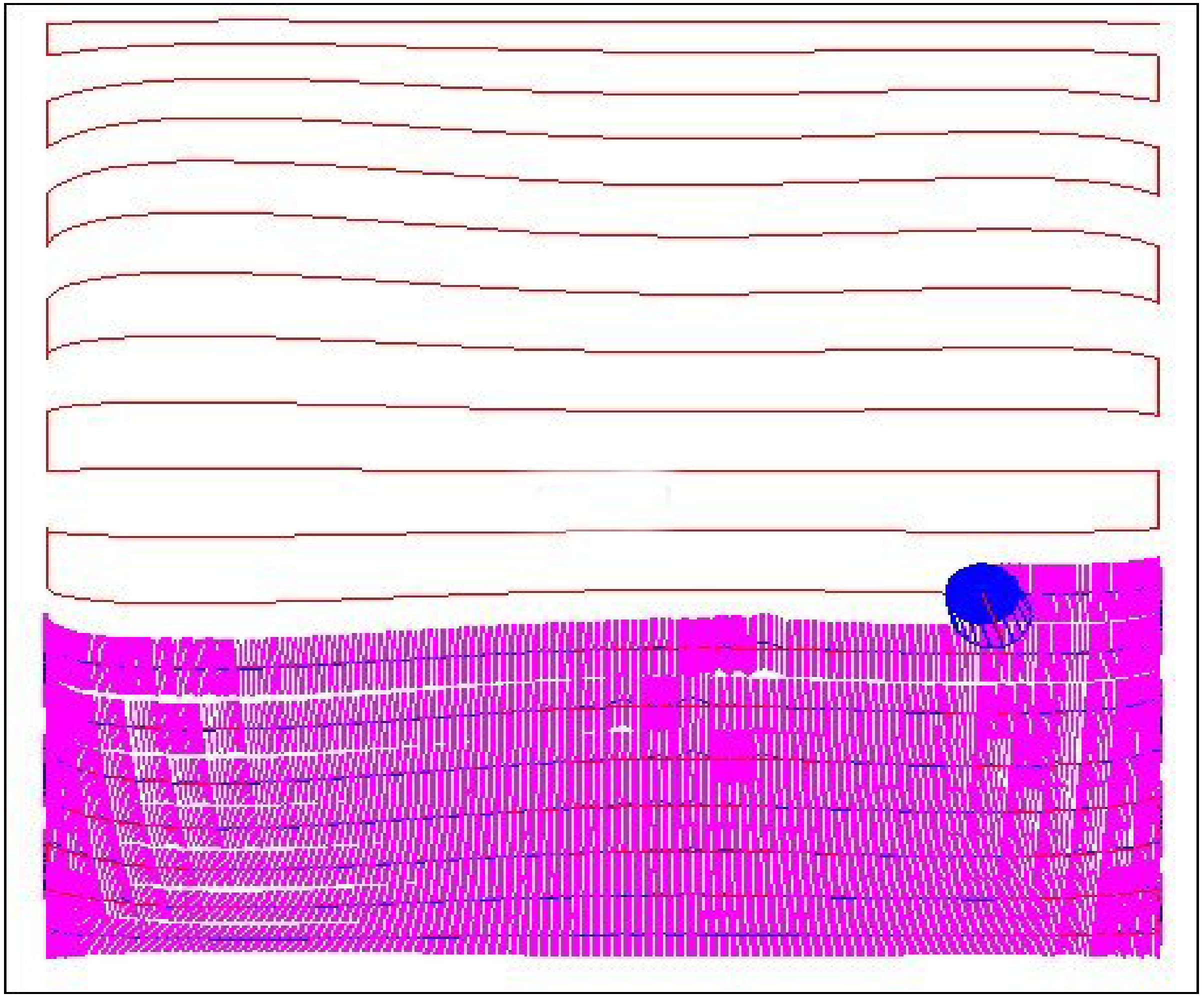


Iso-parametric tool path in v-direction

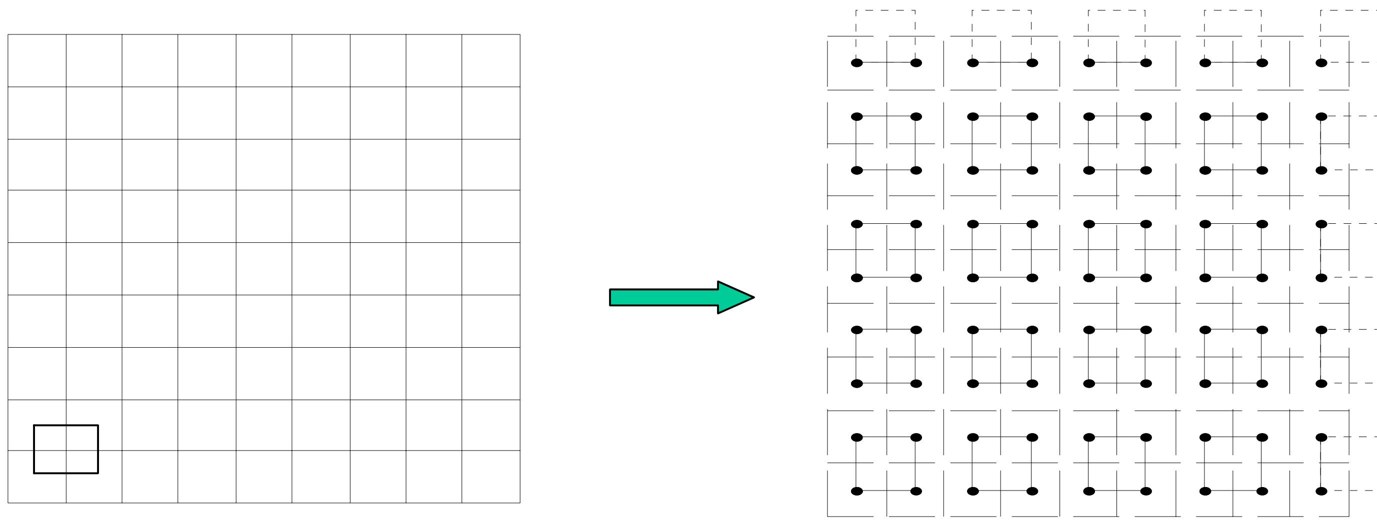
Iso-parametric tool path in u-direction

The resulting grid

Machining strip must cover the entire surface

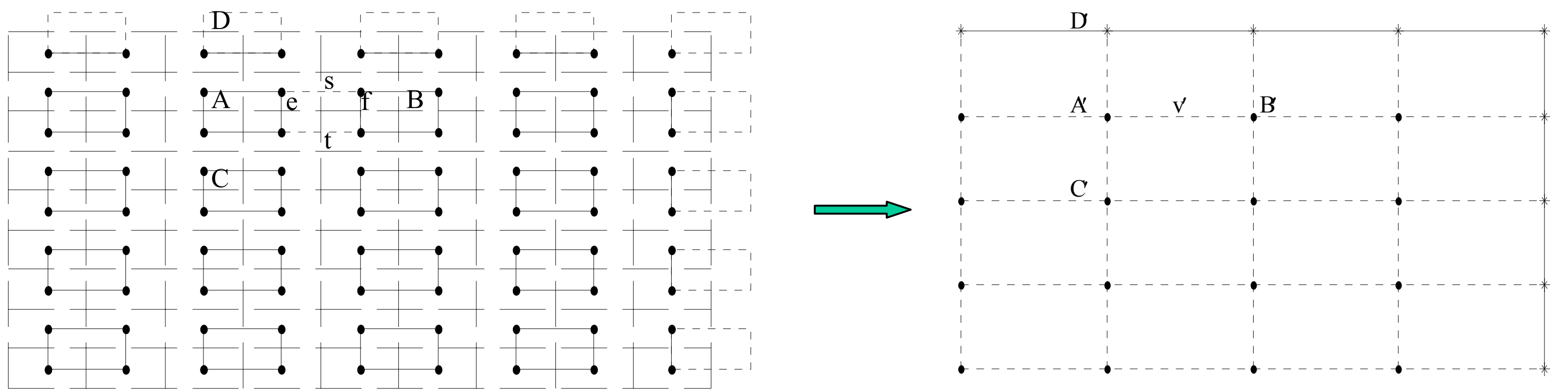


Space-Filling Curve Generation



- Generation of space-filling curve is formulated as the Hamiltonian path problem.
- The grid is first covered by small rectangular circuits.

Space-Filling Curve Generation



Dual Graph G'

- Any two adjacent circuits merge into a bigger circuit. The cost of merging is defined as:

$$Cost(A,B) = |s| + |t| - |e| - |f|$$

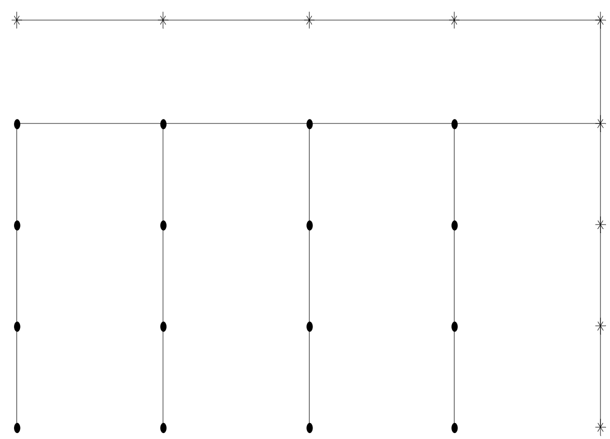
Where |e| presents the length (distance) of the edge e.

- Define a dual graph G':

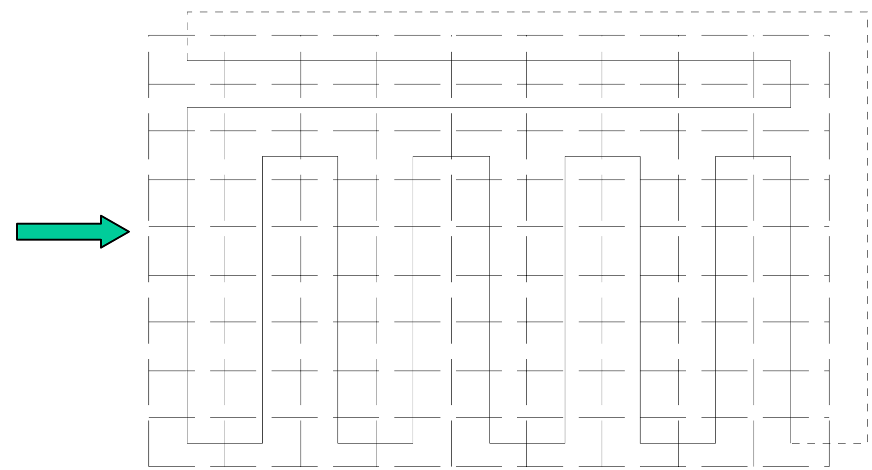
  Each small circuit in G defines a vertex in G'

  Two edges connecting two small circuits in *G* define an edge in G'

Space-Filling Curve Generation



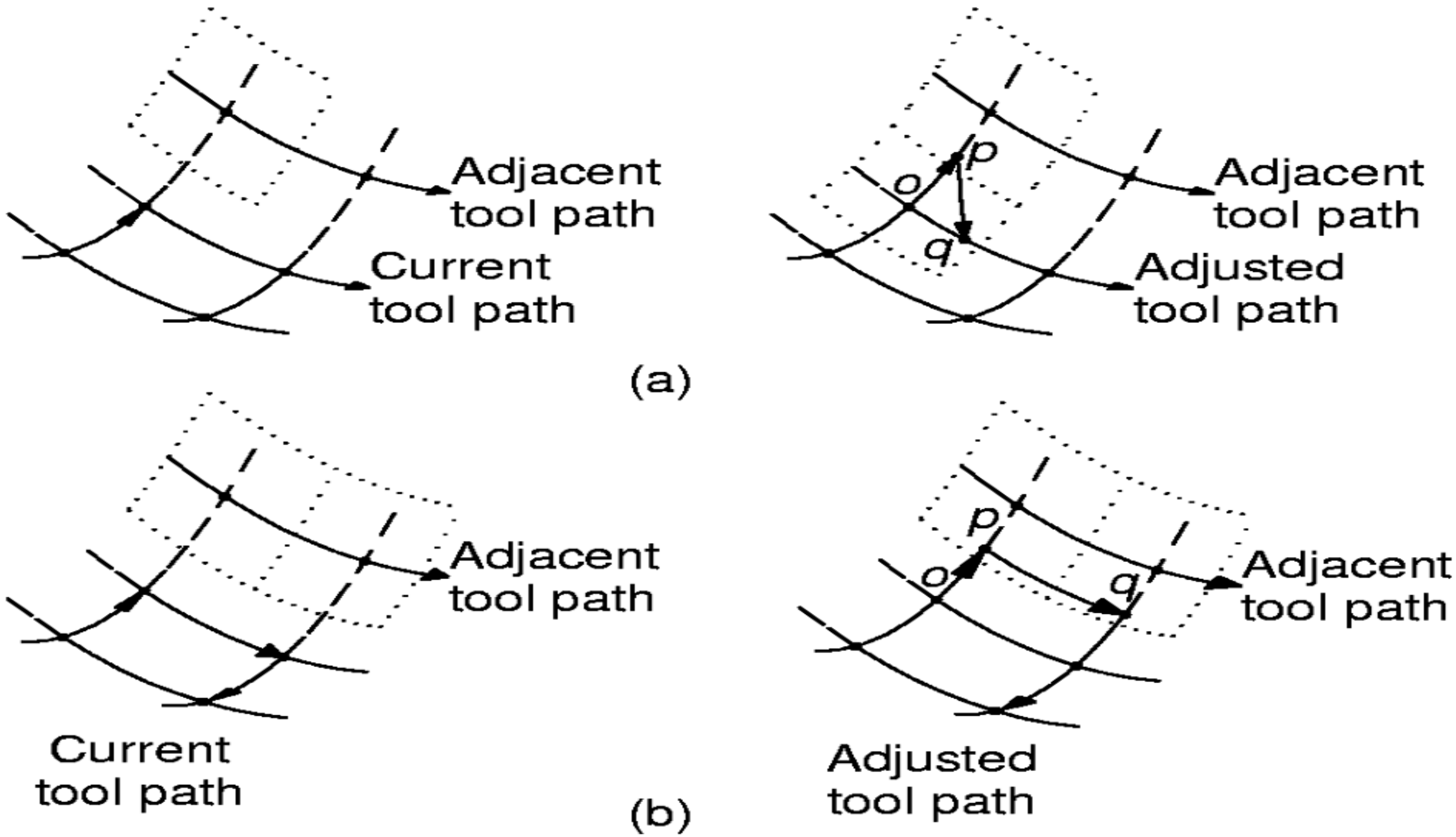
Minimum Spanning Tree
of Dual Graph G'



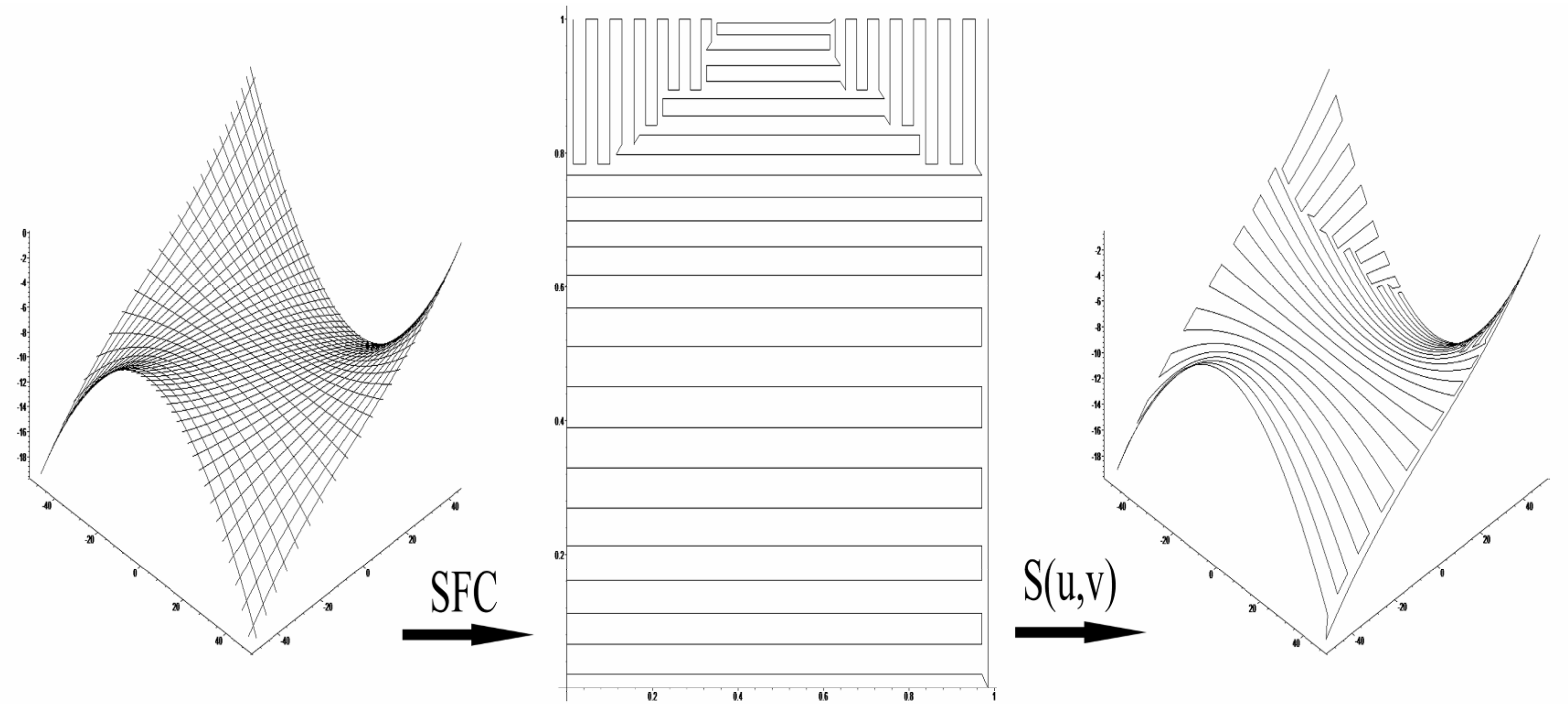
Corresponding Hamiltonian
Path In G

- Merging is done by constructing a minimum spanning tree on the dual grid graph
- Tool path is obtained by removing virtual edges (dashed line), if any.

Tool Path Correction

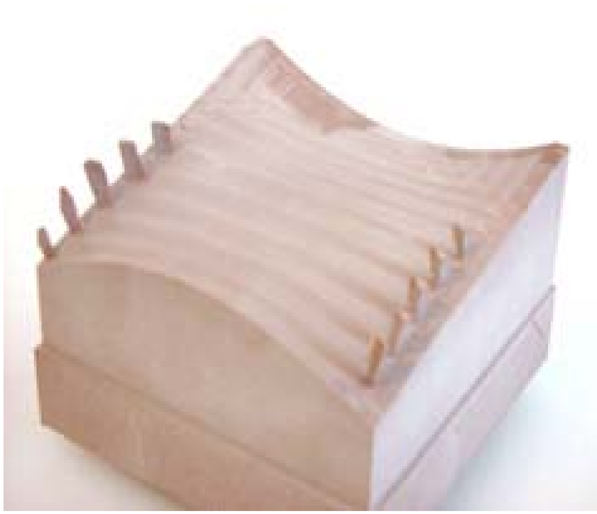


Example 1



(a) (b) (c)

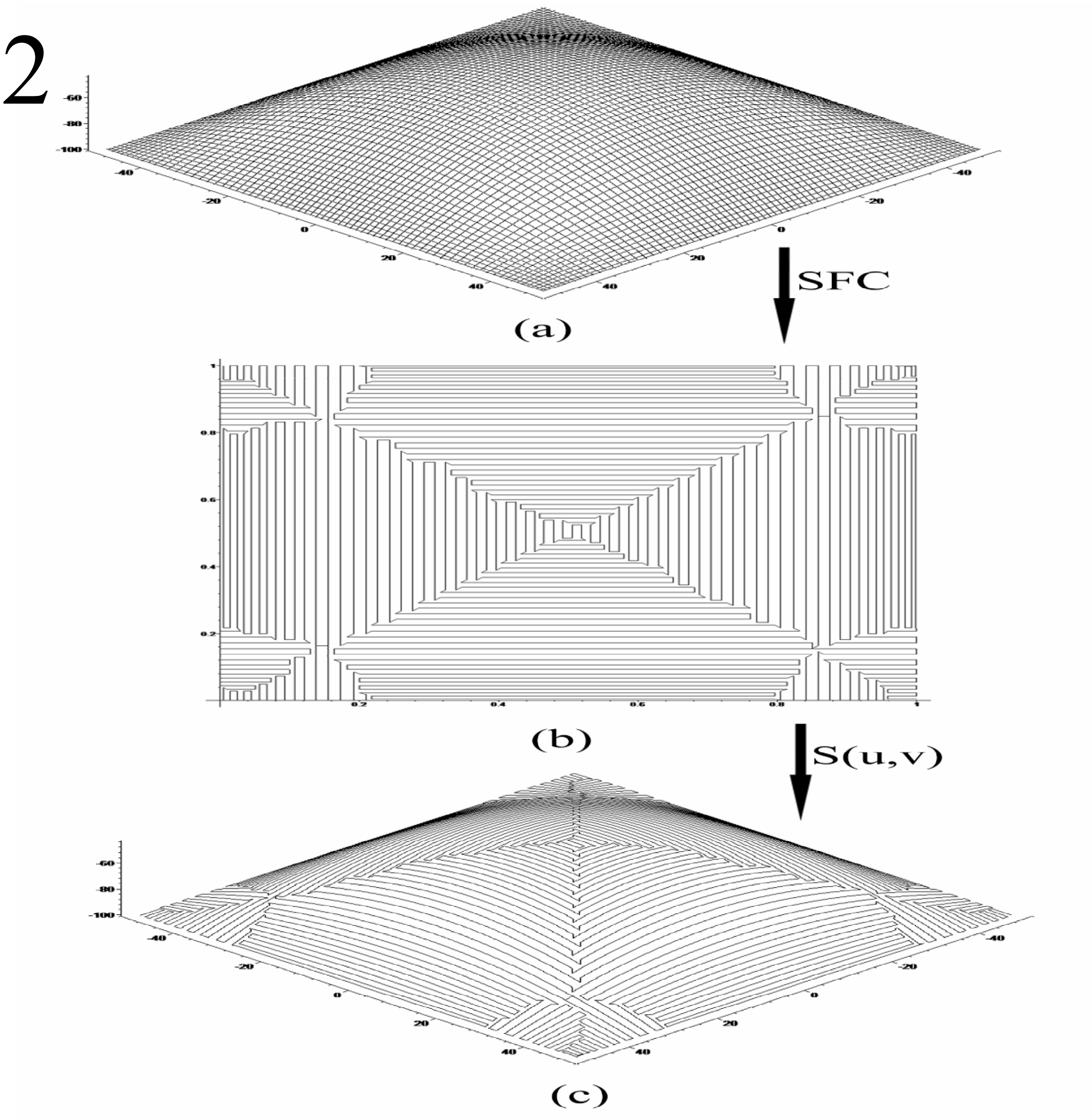
Example 1 Machining



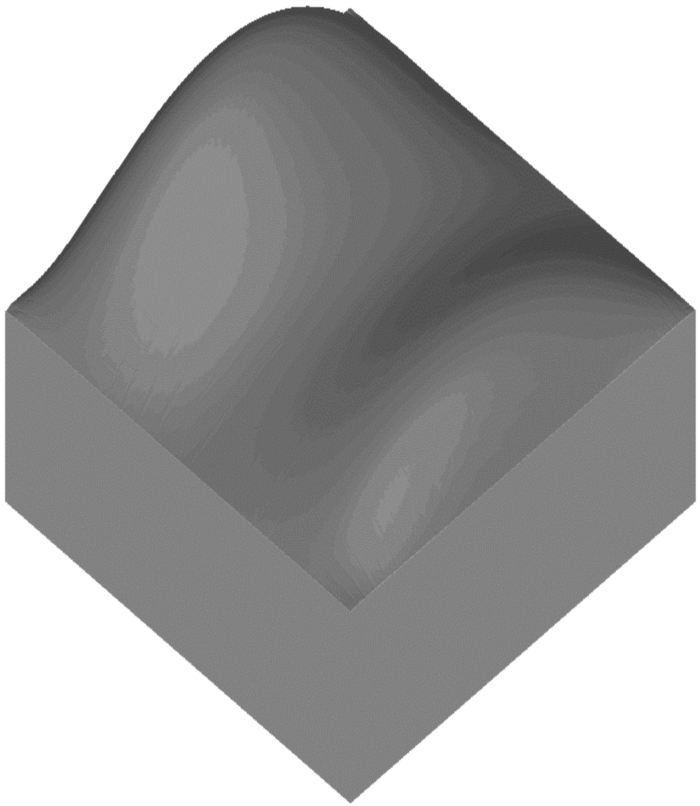
Cutting without tool path correction

Cutting with tool path corrections

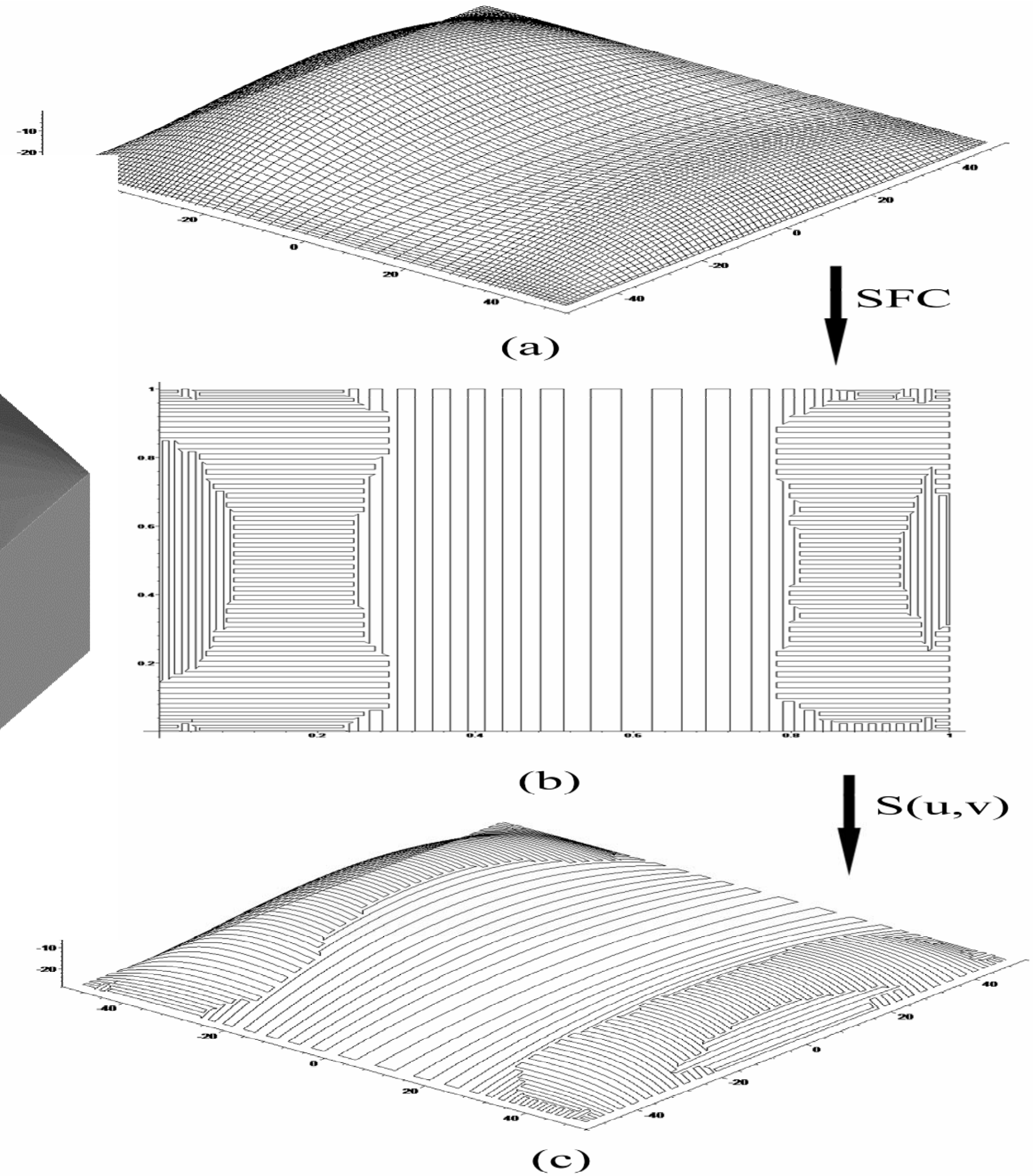
Example 2



Example 3



The Unigraphis Simulator



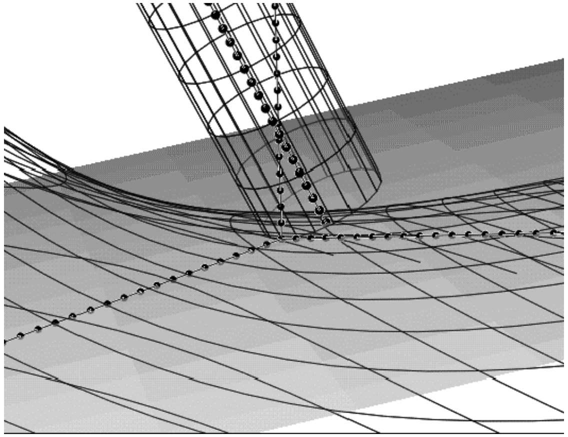
(a)
(b)
(c)

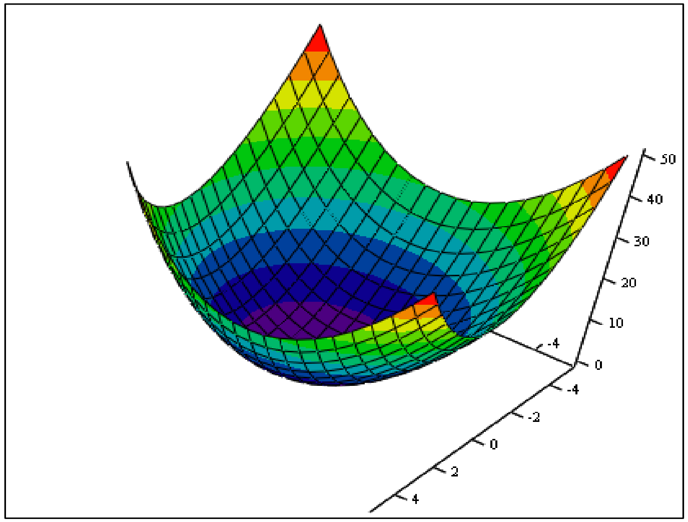
Efficiency of the SFC tool path

Tool Path	Total path length (mm)		
	Example 1	Example 2	Example 3
Iso-parametric in the v-direction	3917.31	9397.97	7831.70
Iso-parametric in the u-direction	2648.12	9397.97	9036.17
SFC tool path	2637.54	7955.58	6780.84

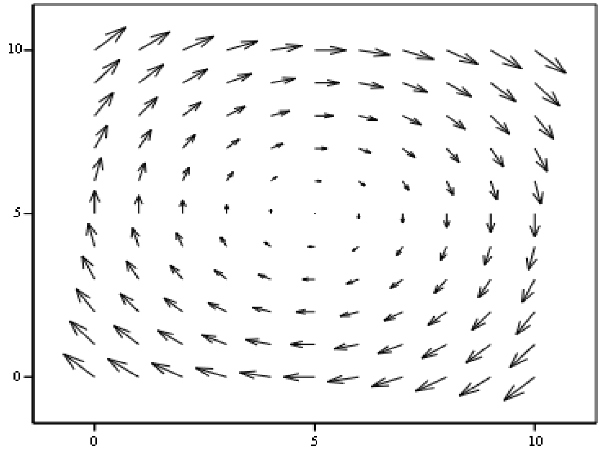
Idea 3. Vector Field Clustering

Calculate optimal directions. Find clusters
of the optimal directions which "look like" zigzag or spiral





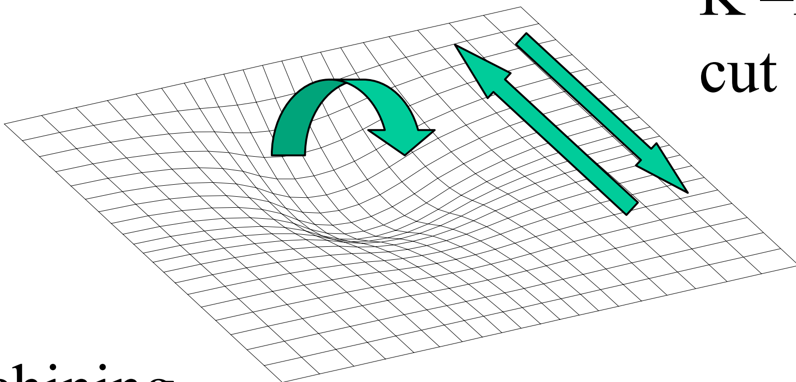
f



(Fy , −Fx)

Vector field analysis

Part surface

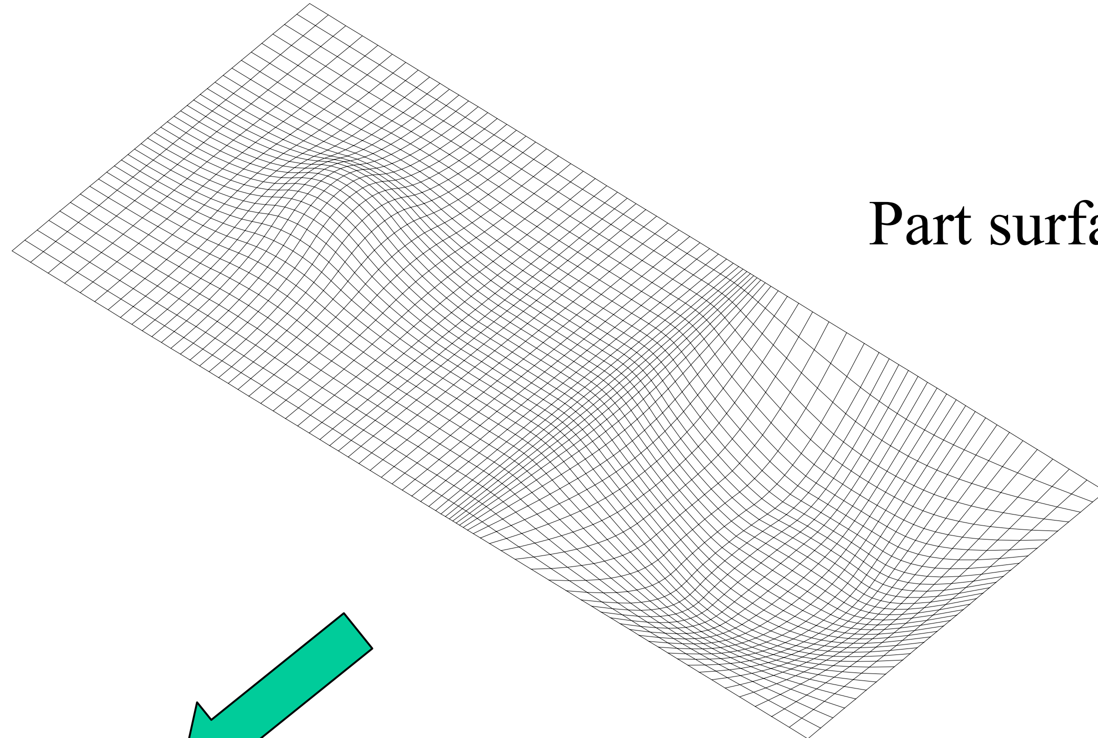


K –means/ Normalized cut method/ DBSCAN

Tool path

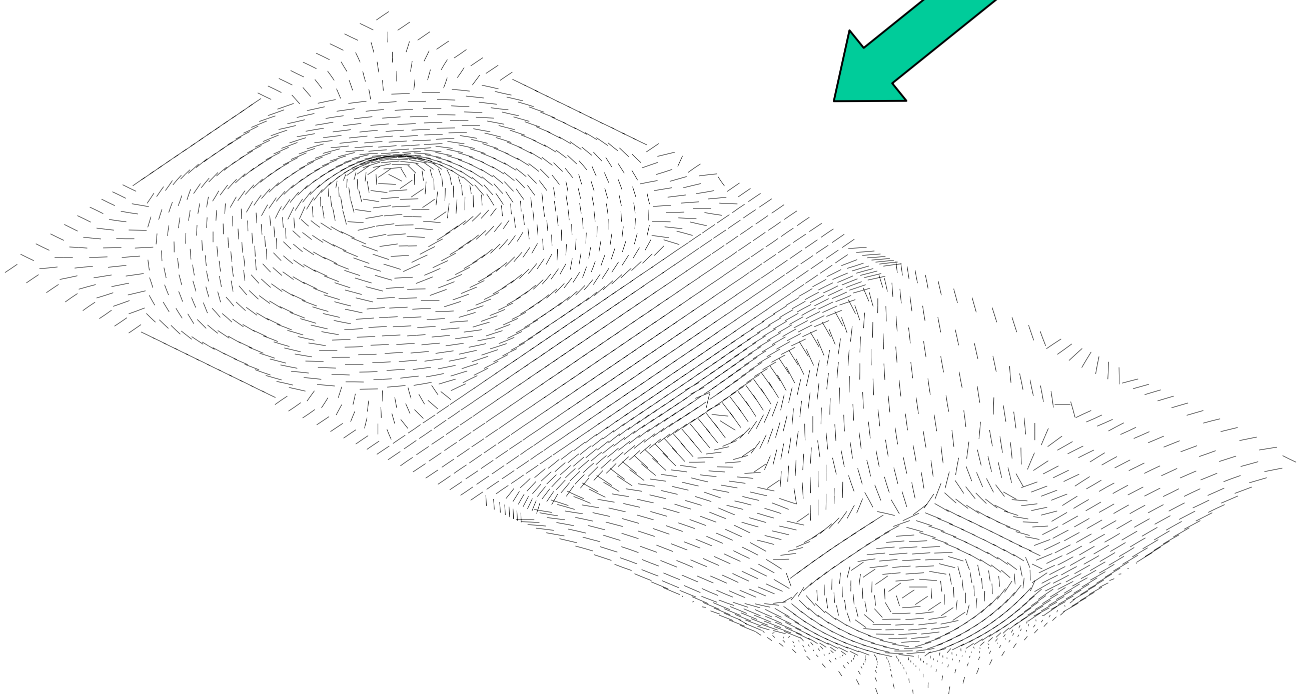
Machining

Example 2



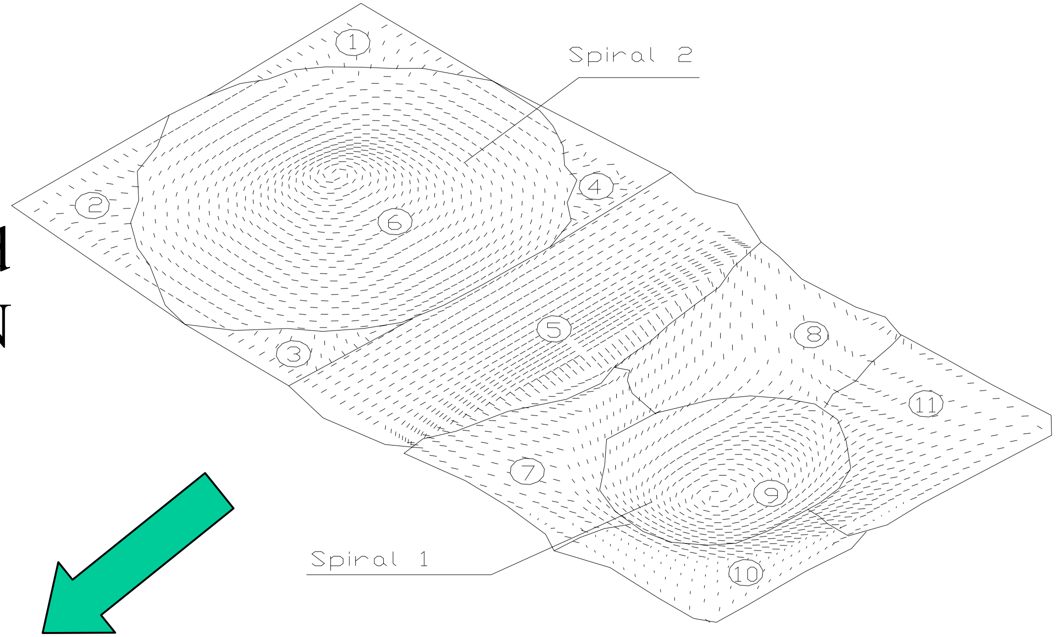
Part surface

Vector field

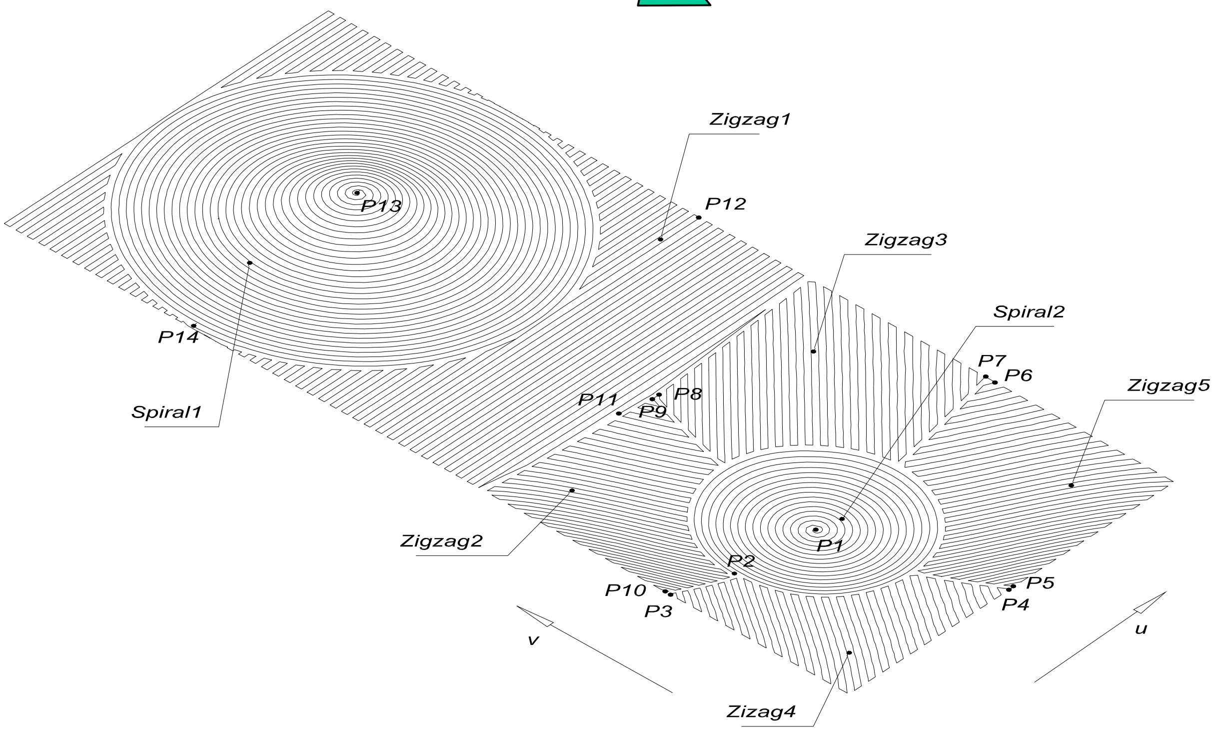


Clusters

K –means/ Normalized cut method/ DBSCAN

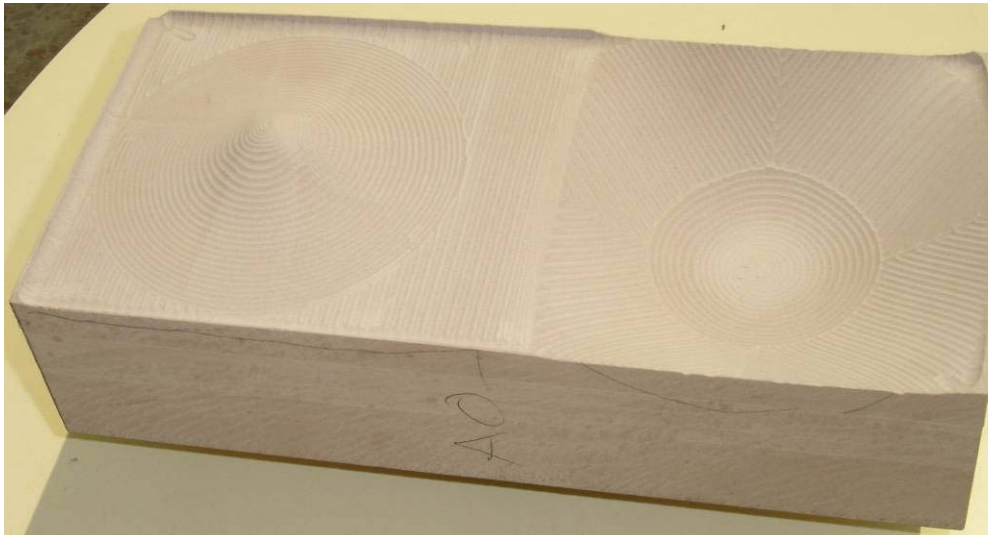


Tool path



Example 2

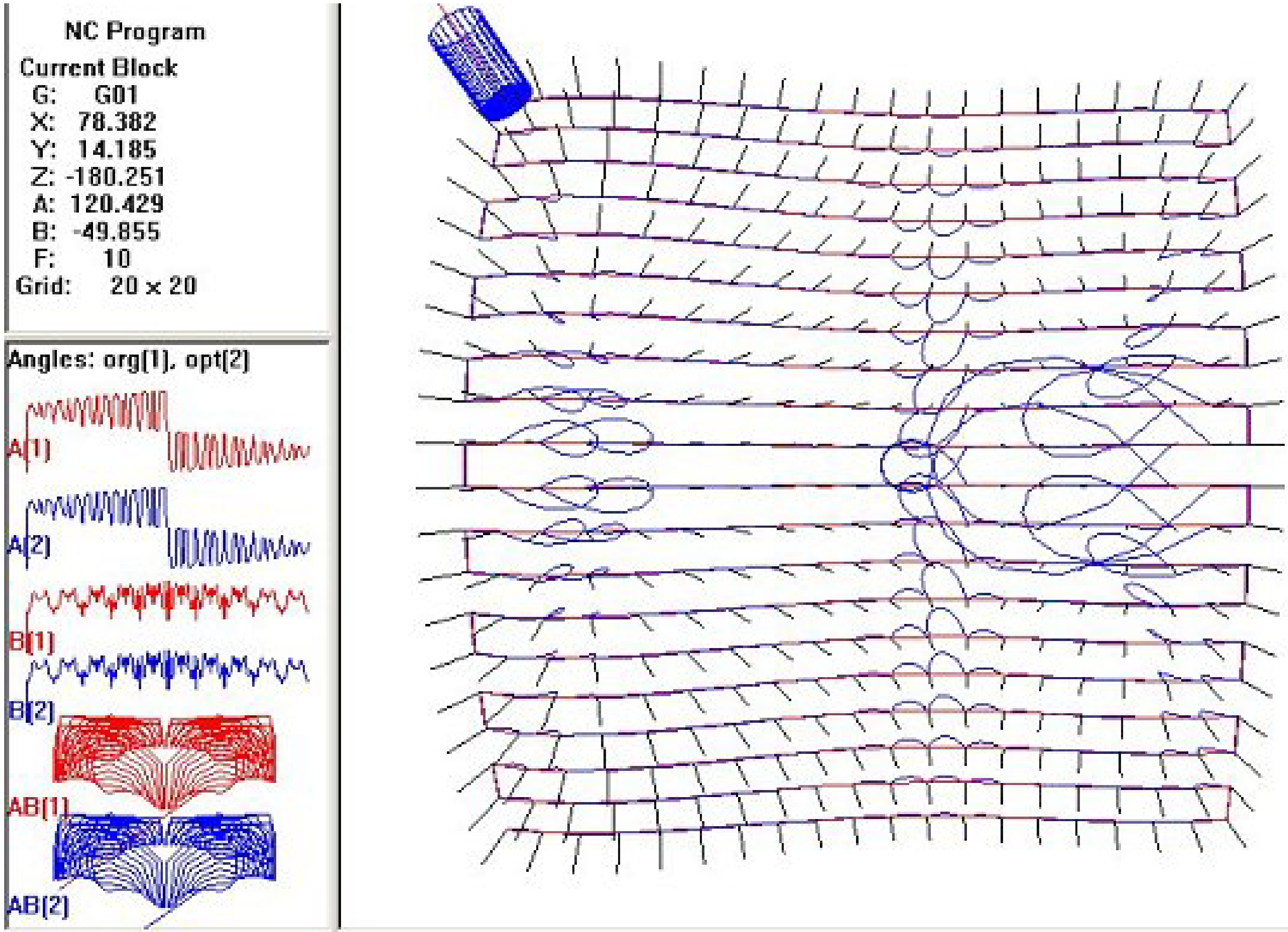
Actual machining



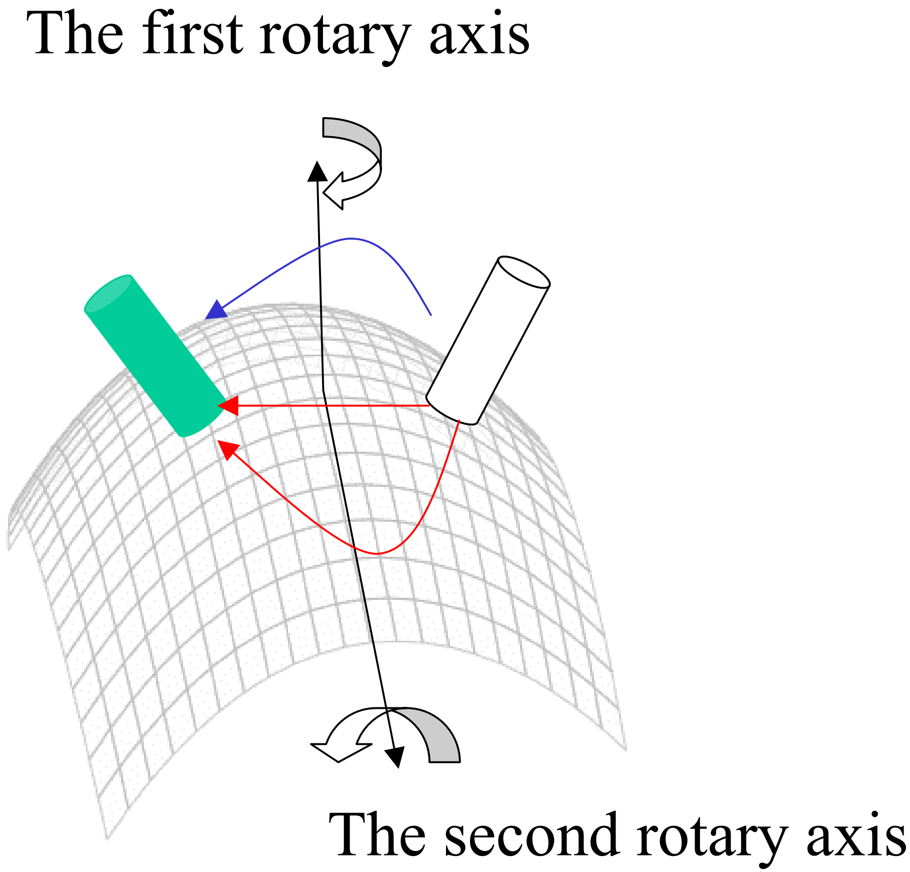
Comparison between tool path calculated by the proposed method and by the iso-parametric method

		CC path length (mm)	Number of turns
Vector Field		18675.45	300
Conventional Method	The *u* dir	20255.37	296
	The *v* dir	20616.16	152

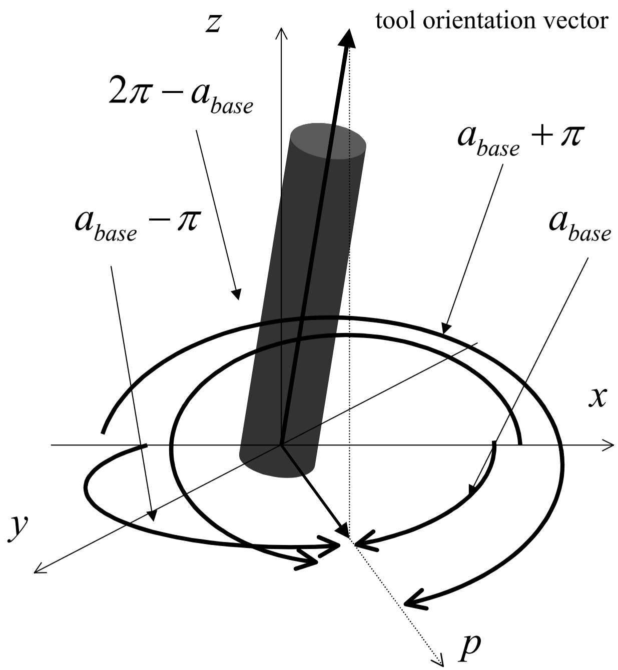
Idea 4. Angle Optimization

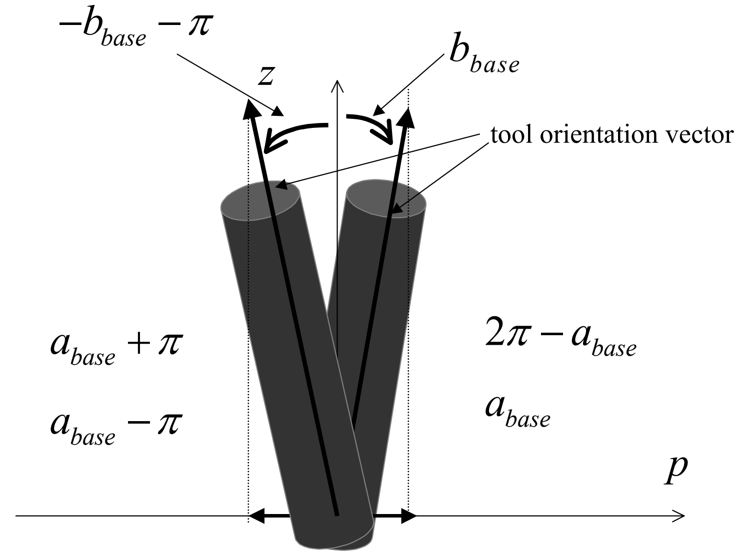


Around or across the hill ?



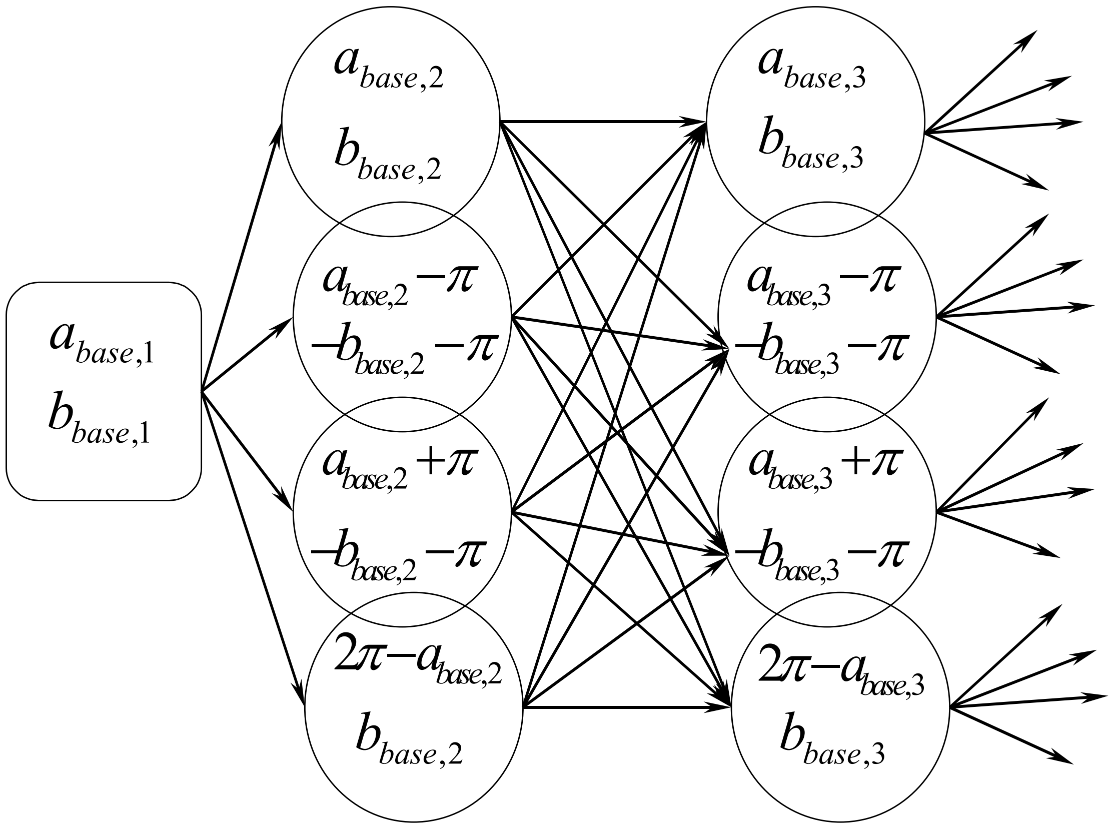
Rotation angles are not unique





$$\Lambda = \begin{cases} a_{base}, b_{base} \\ a_{base} - 2\pi, b_{base} \\ a_{base} - \pi, -b_{base} - \pi \\ a_{base} + \pi, -b_{base} - \pi. \end{cases}$$

The Shortest Path



The shortest path for 2 points

Before:

$$a_1 = -1.571, b_1 = -0.896,$$

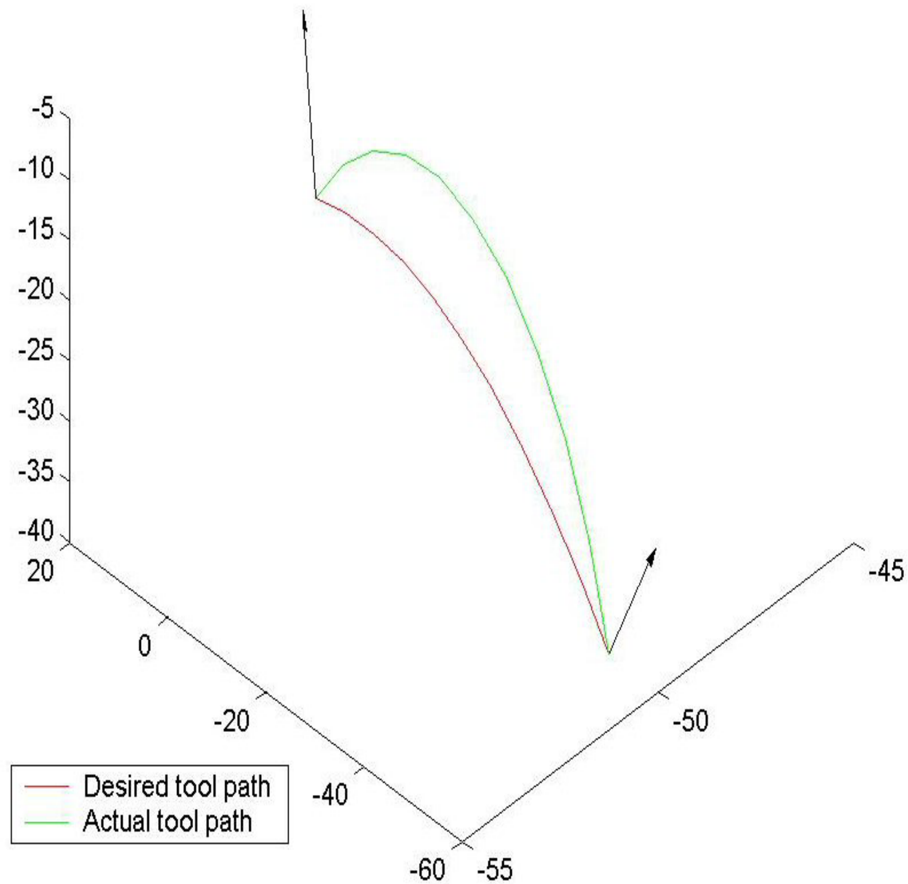
$$a_2 = -4.712, b_2 = -1.412.$$

After:

$$a_2' = a_2 + \pi, b_2' = -\pi - b_{2,}$$

$$a_1 = -1.571, b_1 = -0.896,$$

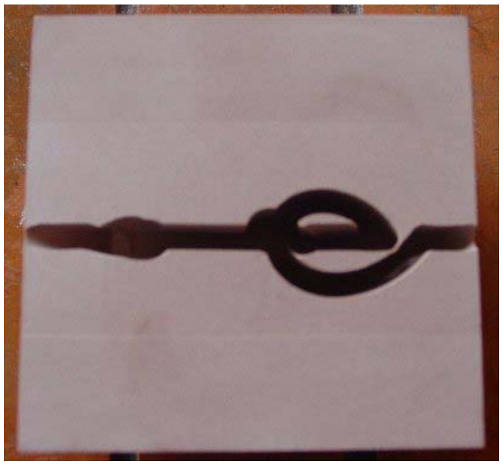
$$a_2' = -1.571, b_2 = -1.729.$$

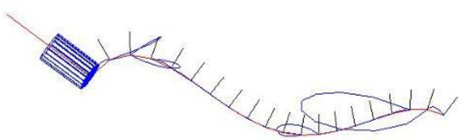


$$\text{Error Reduction} = 98.25\%$$

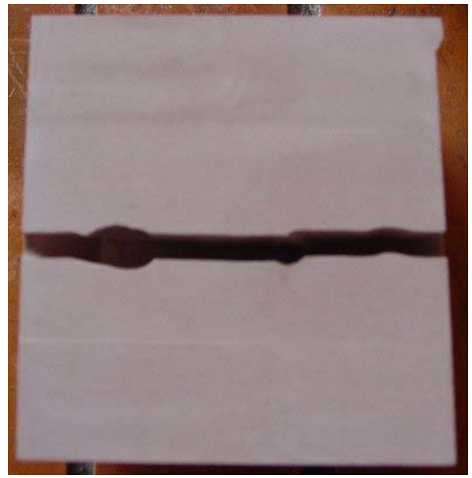
Before and after

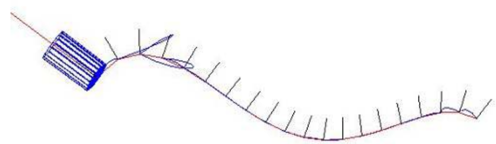
Before optimization



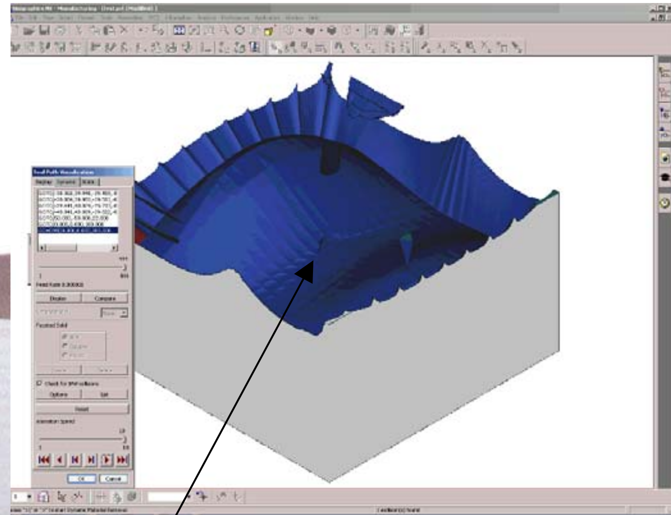


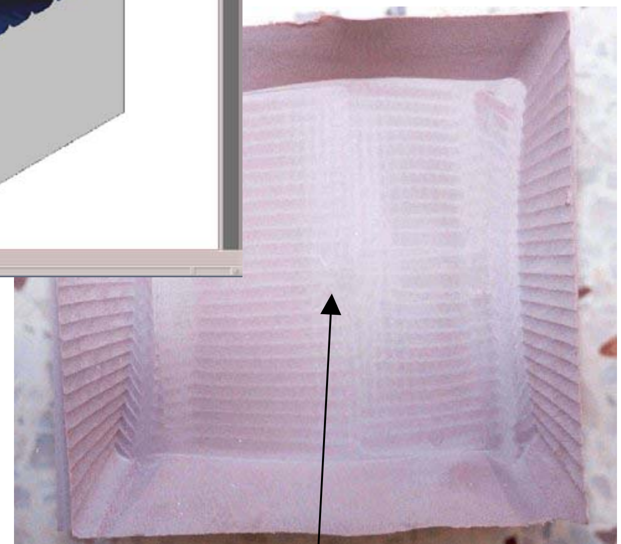
After optimization

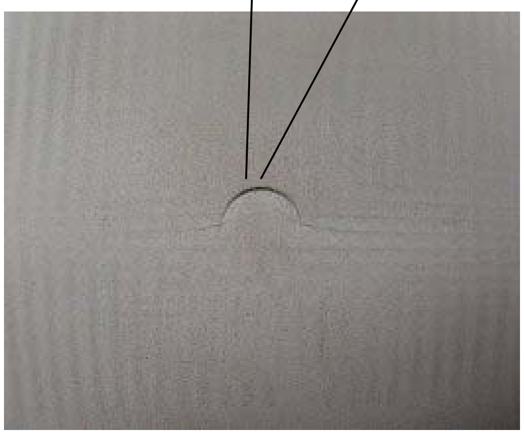


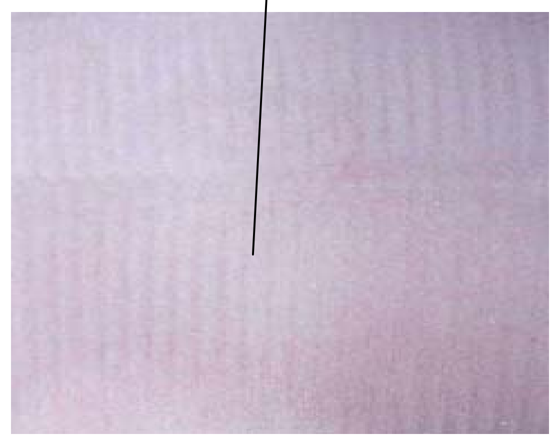


Before and After





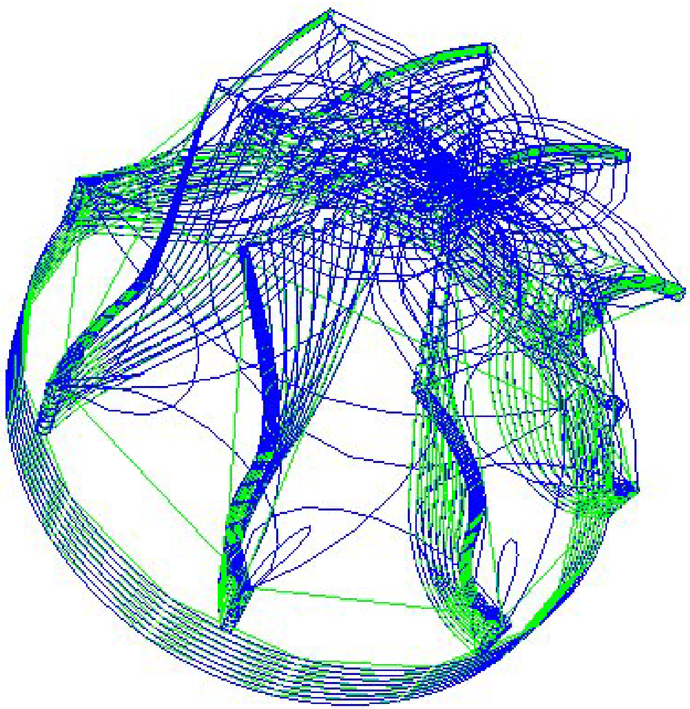


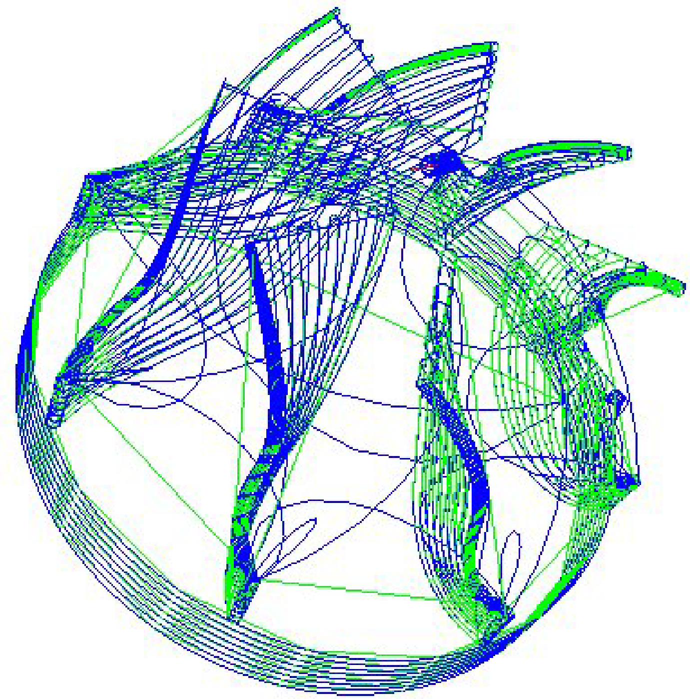


Efficiency

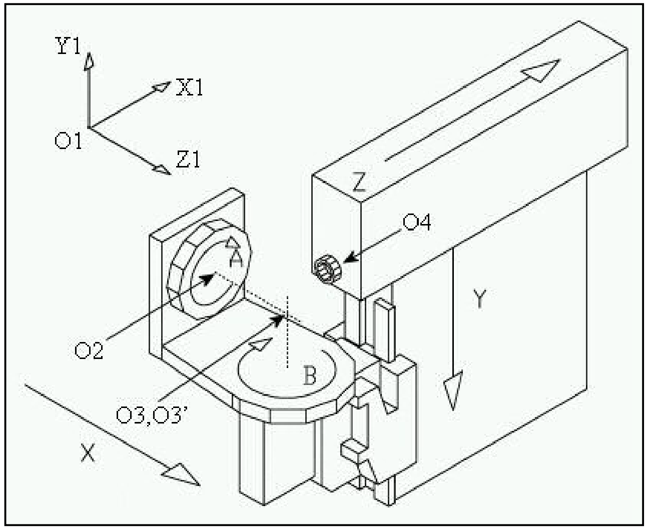
Tool path	No optimization max error(mm)	Optimization max error (mm)	error decrease (%)	Path length Non-opt/opt(mm)
10 x 20	23.862	12.426	47.9	2825.6 / 2255.5
15 x 20	19.300	8.517	55.9	2500.3 / 2123.0
20 x 20	20.228	7.558	62.6	2367.6 / 2101.2
30 x 20	16.253	7.162	56.0	2183.6 / 2038.3
40 x 20	8.711	6.878	21.0	2069.3 / 2020.1
100 x 20	7.395	7.103	4.0	1916.1/ 1911.2
130 x 20	3.999	3.999	0	1876.49 / 1876.49

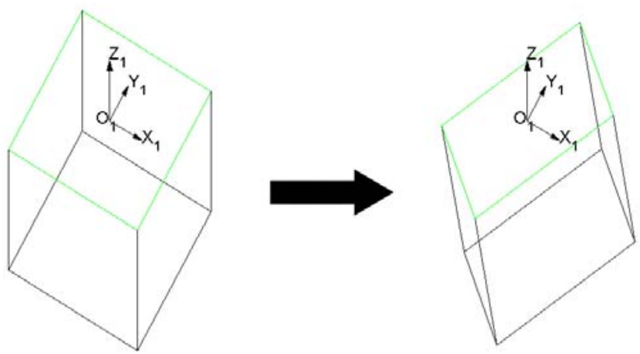
Impeller before and after

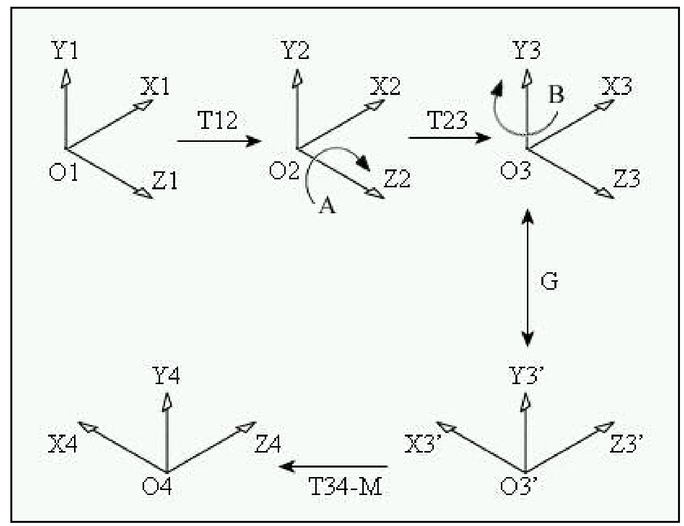




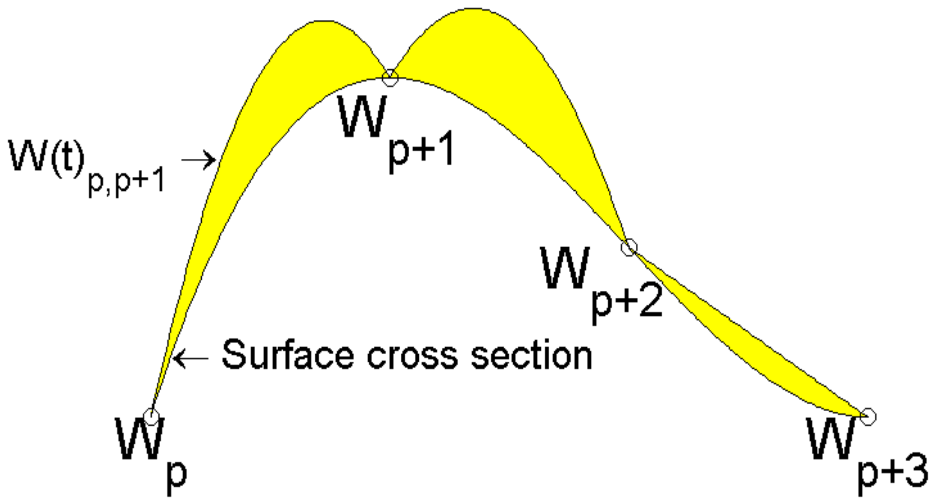
Idea 5. The error depends the on workpiece initial orientation and the machine setup







Least Square Error



$$\varepsilon = \sum_p \int_0^1 \left| W^D_{p,p+1} - W_{p,p+1} \right|^2 dt,$$

$$= \sum_p \int_0^1 (x^D_{p,p+1} - x_{p,p+1})^2 + (y^D_{p,p+1} - y_{p,p+1})^2 + (z^D_{p,p+1} - z_{p,p+1})^2 dt.$$

System of nonlinear Equations

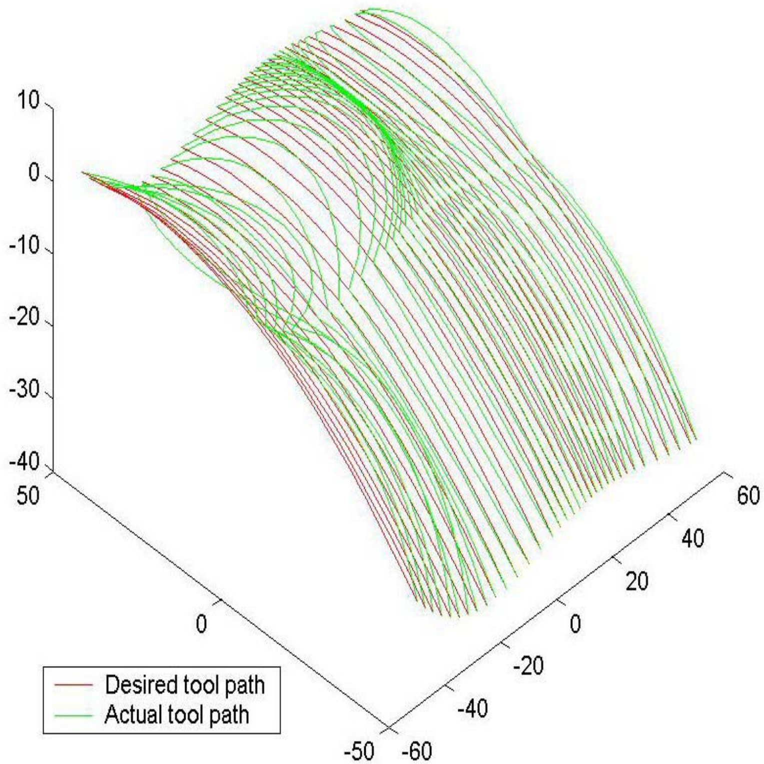
- Differentiate the error function with respect to workpiece (R_a, R_b, T_{12}),
- Solved by Newton Method.

The Jacobian matrix for the case

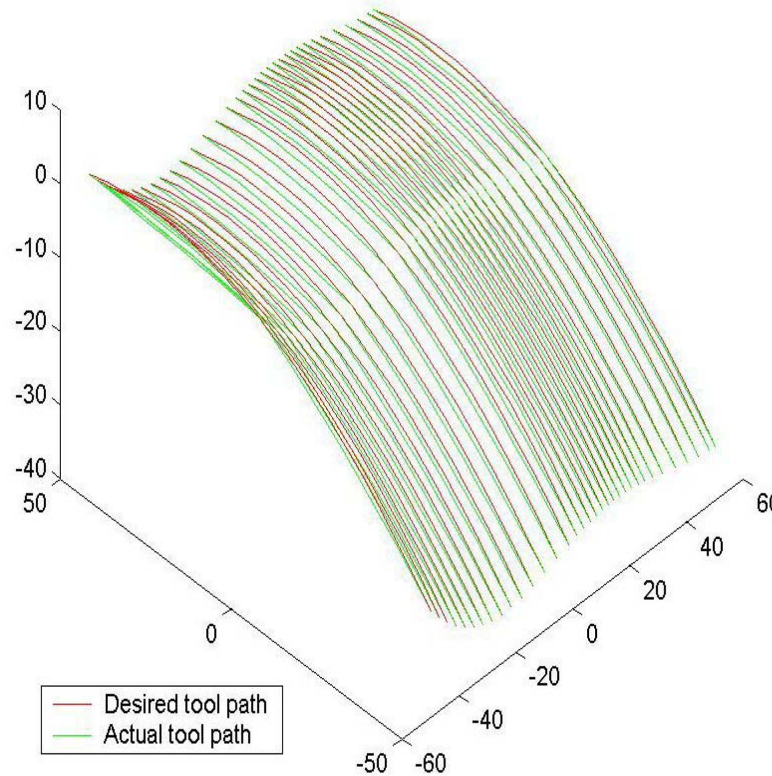
R_a, R_b, T_{12}

$$J = \begin{bmatrix} \frac{d^2\varepsilon}{dR_a^2} & \frac{d^2\varepsilon}{dR_a dR_b} & \frac{d^2\varepsilon}{dR_a dT_{12}} \\ \frac{d^2\varepsilon}{dR_a dR_b} & \frac{d^2\varepsilon}{dR_b^2} & \frac{d^2\varepsilon}{dR_b dT_{12}} \\ \frac{d^2\varepsilon}{dR_a dT_{12}} & \frac{d^2\varepsilon}{dR_b dT_{12}} & \frac{d^2\varepsilon}{dT_{12}^2} \end{bmatrix}$$

Example 1 Tool Path

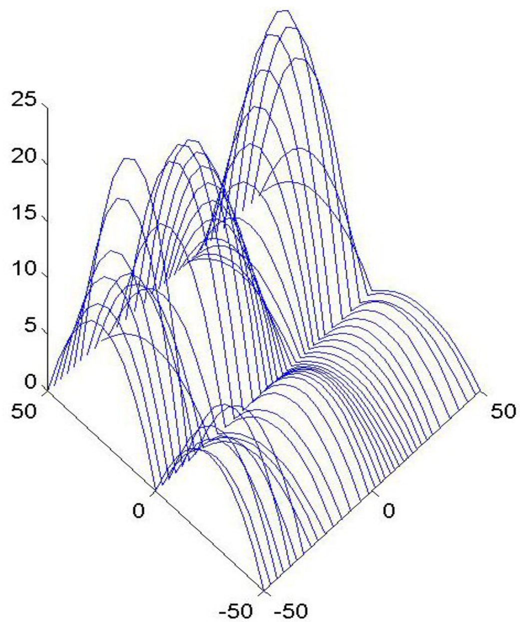


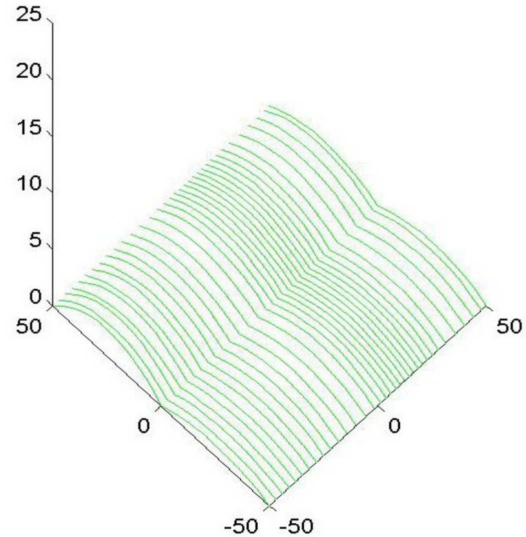
Before optimization



After optimization

Example 1 Error





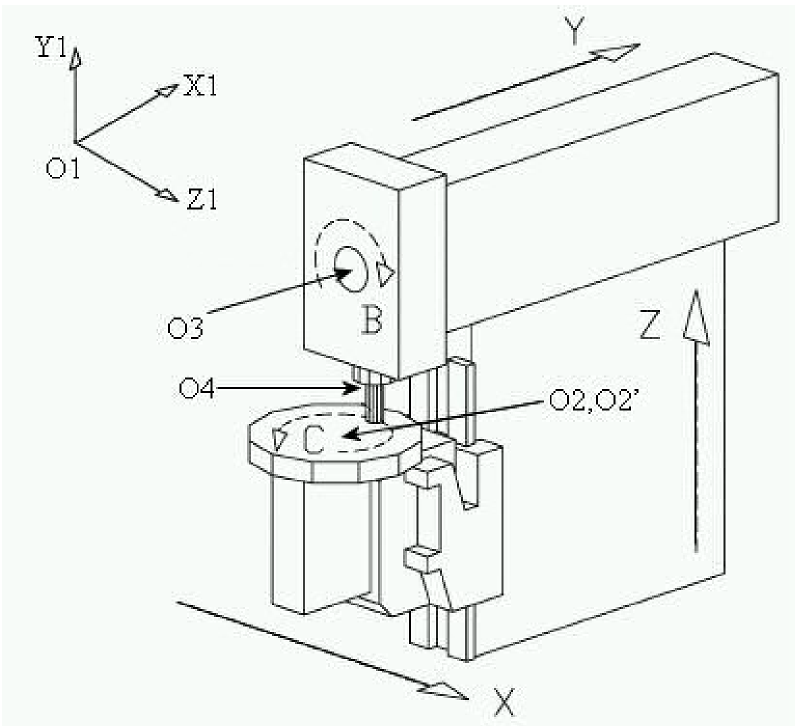
$\text{Reduction} = 98.20\%$

Before optimization

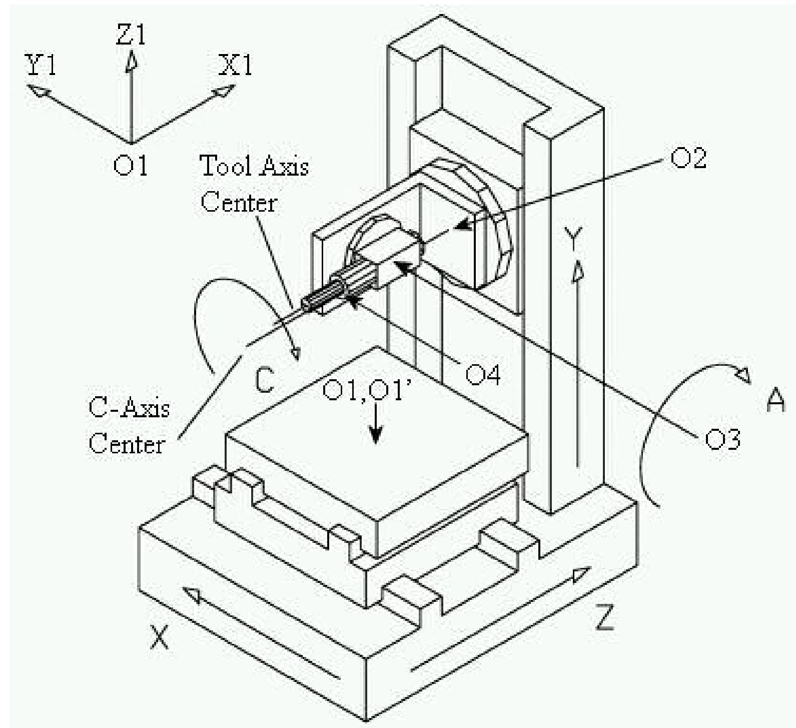
After optimization

Other machines Similar results

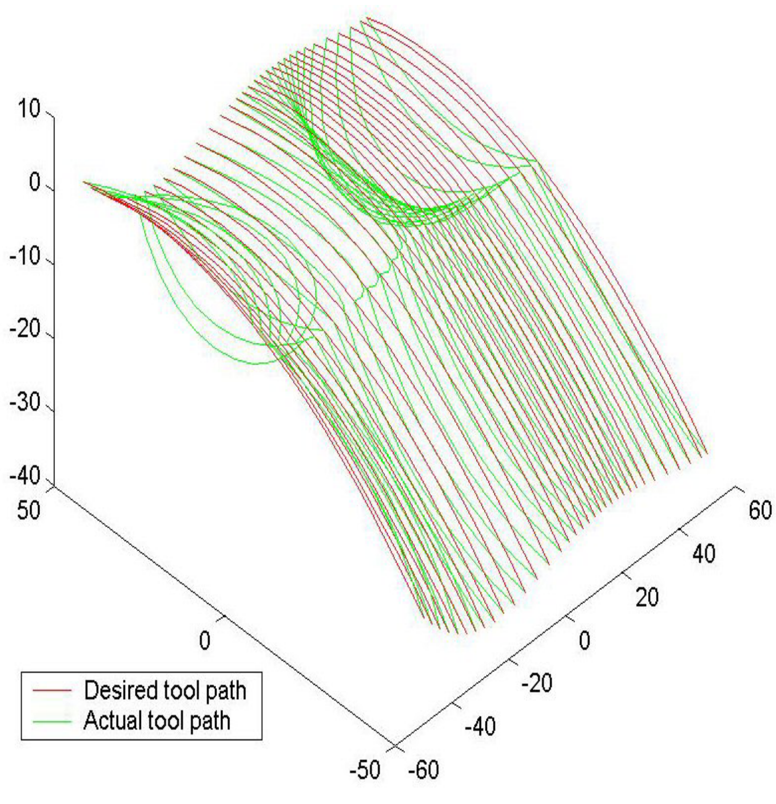
The 1-1 machine



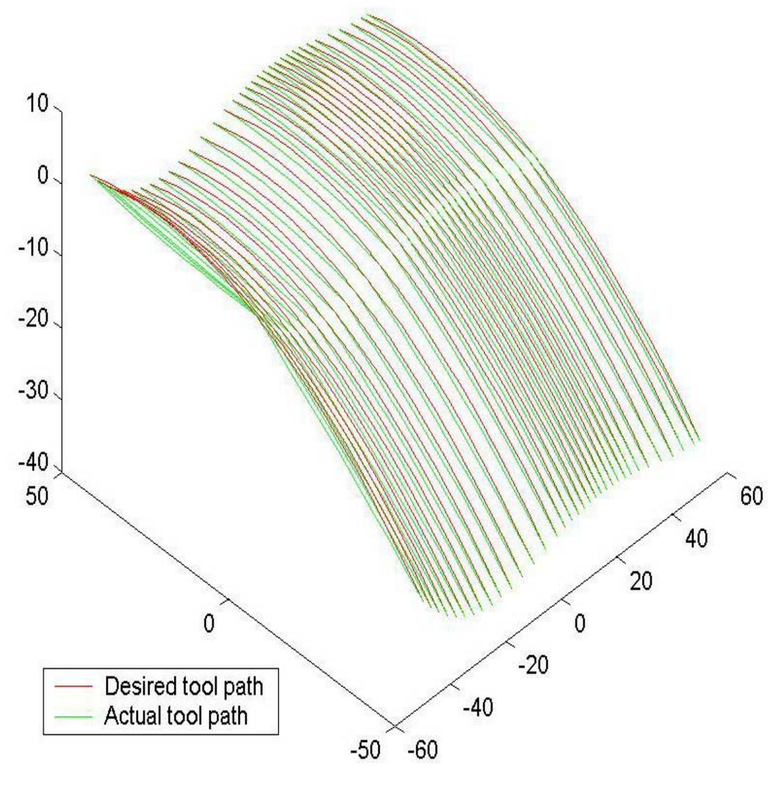
The 0-2 machine



Example 2. The 1-1 Machine

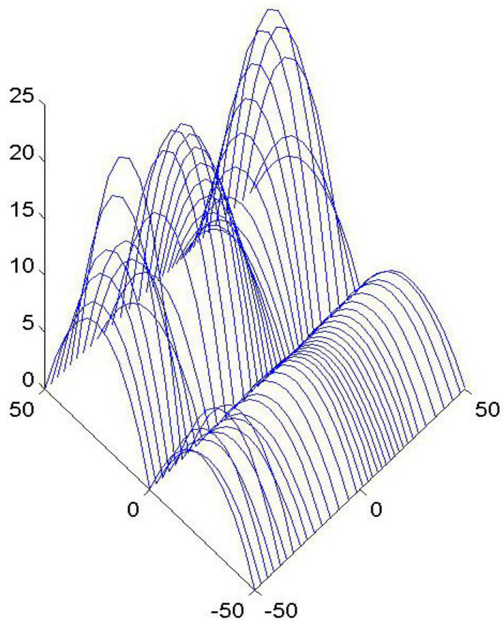


Before optimization

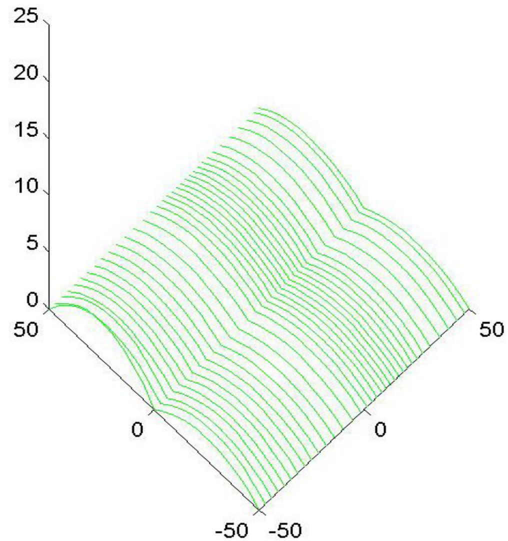


After optimization

Example 2. The 1-1 Machine



Before optimization



After optimization

$$\boxed{\text{Reduction} = 98.03\%}$$

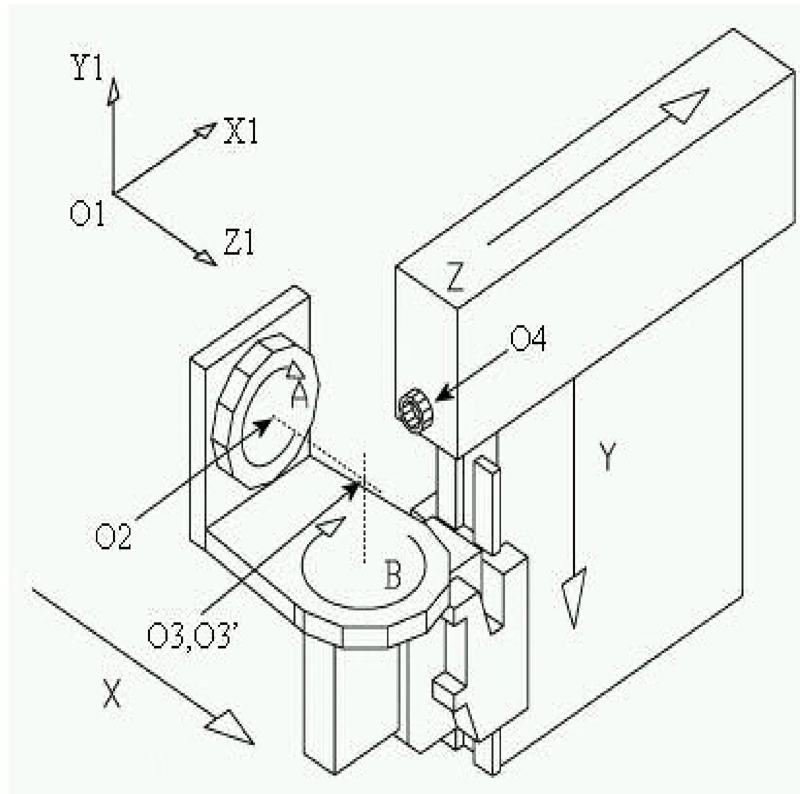
We obtained a rigorous mathematical proof that every machine has 6 optimizable variables

Machine Type	Workpiece setup	Tool	Machine settings
The 2-0 machine	$r_a, r_b, T_{12,x}, T_{12,y}, T_{12,z}$	none	$T_{23,x}$
The 1-1 machine	$r_a, r_b, T_{12,x}, T_{12,y}$	$T_{4,z}$	$T_{34,x}$
The 0-2 machine	r_a, r_b	$T_{4,z}$	$T_{23,x}, T_{23,y}, T_{34,y}$

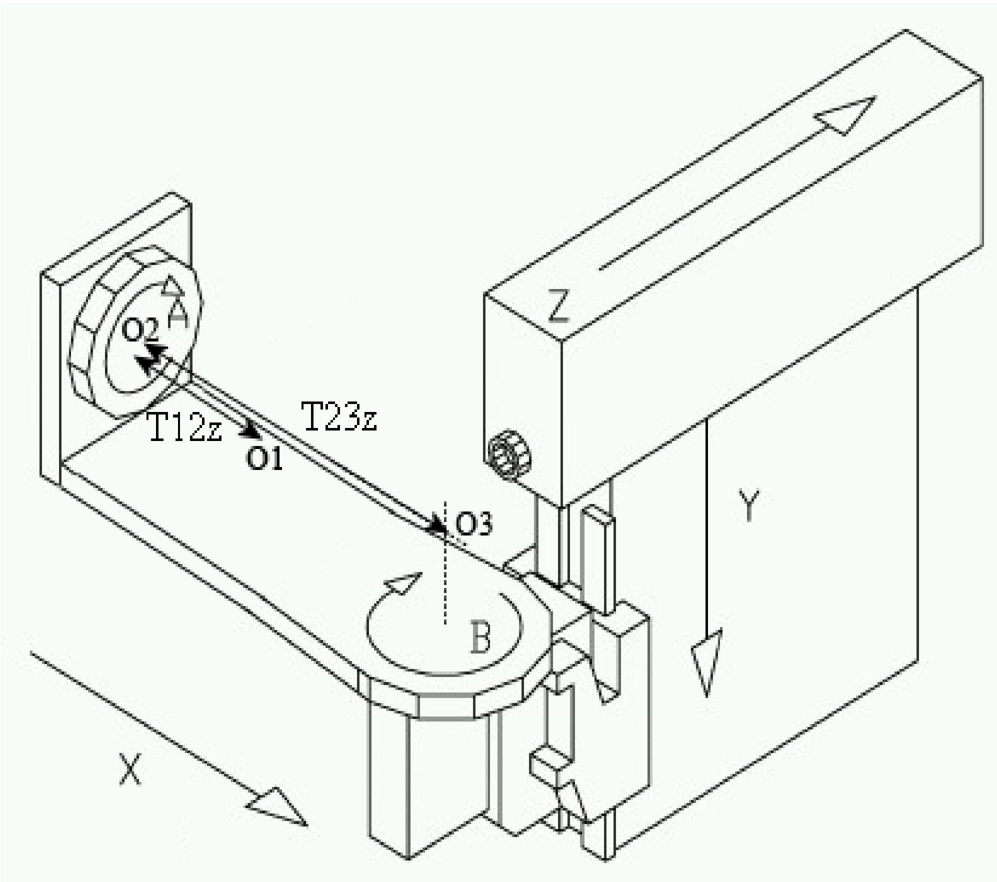
Other parameters are either invariant parameters

(The parameter doesn't affect the tool trajectory. For example: the tool length for the 2-0 machine)

$$\frac{d\varepsilon}{dv} \equiv 0$$



or dependent variables

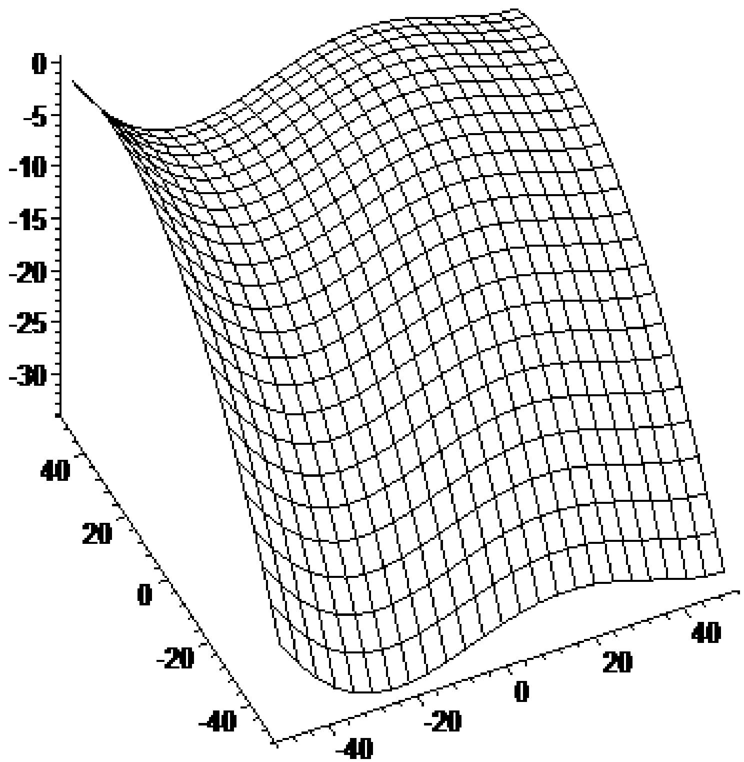


$$T_{12,z} + T_{23,z} = c$$

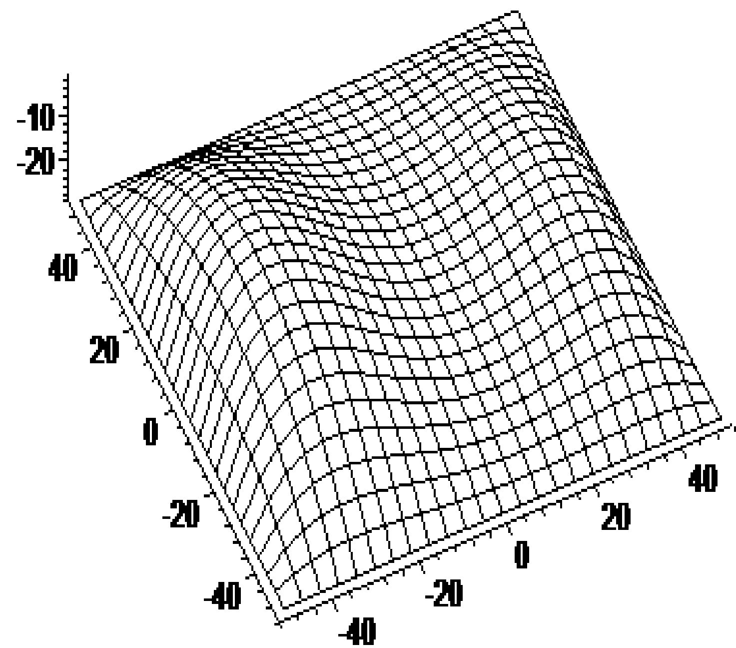
$$T_{13,z} = c$$

Efficiency

Surface 1



Surface 2



Efficiency Surface 1

Machine type	mean error (mm) / Reduction			
	Before	Optimization workpiece setup	Optimization wrt. the workpiece setup and the tool length	Optimization wrt. the entire set
The 2-0 machine	0.5730	0.0179 / 96.88%		0.0176 / 96.92%
The 1-1 machine	0.5785	0.0360 / 93.77%	0.0186 / 96.78%	0.0186 / 96.78%
The 0-2 machine	0.0367	0.0355 / 3.13%	0.0200 / 45.36%	0.0179 / 51.11%

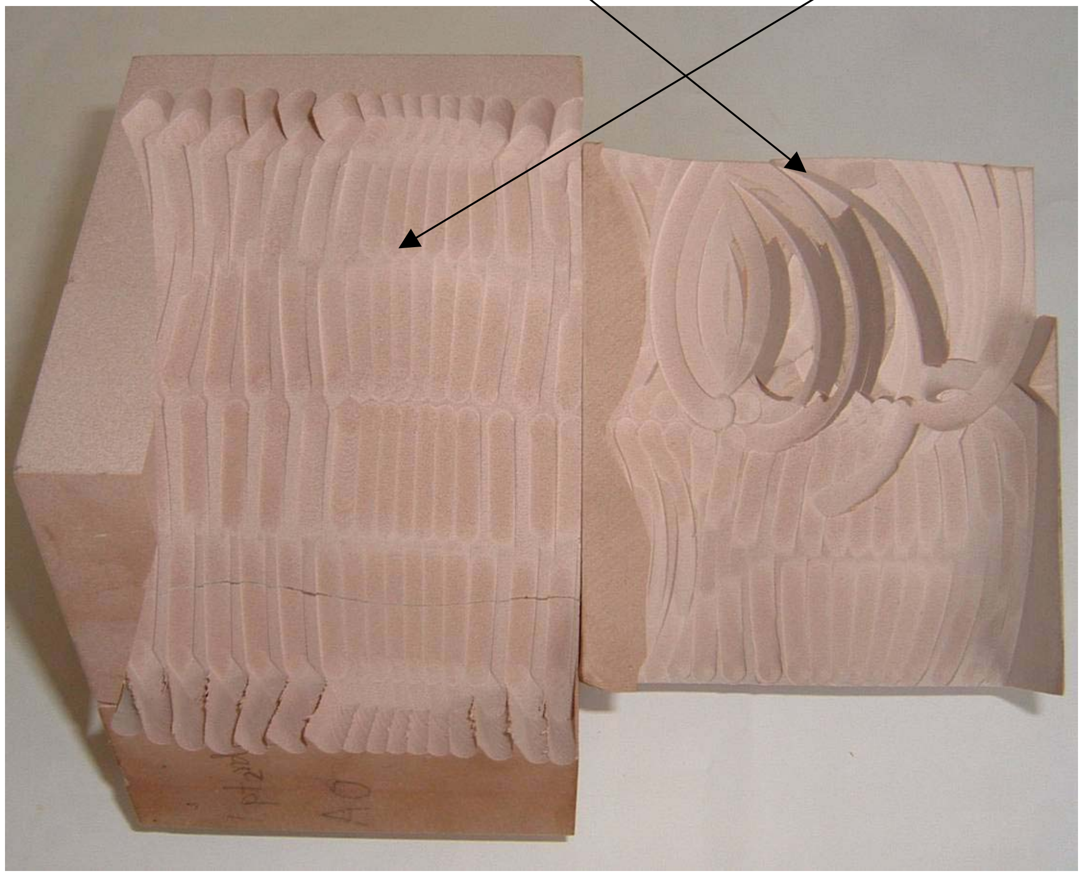
Efficiency Surface 2

Machine type	mean error (mm) / Reduction			
	Before optimization	Optimization wrt. the workpiece setup	Optimization wrt. the workpiece setup and the tool length	Optimization wrt. the entire set
The 2-0 machine	0.5408	0.1118 / 79.33%		0.1102 / 79.62%
The 1-1 machine	0.7092	0.6335 / 10.68%	0.3072 / 56.68%	0.3029 / 57.30%
The 0-2 machine	0.5561	0.5495 / 1.18%	0.1220 / 78.06%	0.1087 / 80.45%

Efficiency. Point Reduction

Surface	# of CC points / Reduction	
	Before optimization	Optimization wrt. r_a and r_b
Surface 1	3900	1233 / 68.4%
Surface 2	7925	5544 / 30.0%

Machining. The 2-0 machine. Surface 1 Before and After



Conclusions

We proposed and analyzed 5 methods to optimize five-axis machining

1) Curvilinear grid techniques (30-40%) accuracy increase
2) Space filling curve 10-30% tool path length decrease
3) Vector field clustering 10-20% tool path length decrease
4) The shortest path minimization with regard to the angles up to 90% accuracy increase for rough cut
5) The Least square minimization with regard to the initial position, orientation of the workpeice and configuration of the machine up to 90% accuracy increase. 30-60% decrease in the number of points (positions)

A New Software for 5-axis Machining, Optimization, Simulation, and Verification Research Team

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Visualization , Solid Modeling

Optimization,
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CIM Lab Manager, AIT

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machining

Master student (vacant)
With the TA-ship

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