Robust Stabilization of State Delayed System

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ABSTRACT – In this paper, we present a method of stabilizing uncertain time-delay systems. The systems under consideration are described by linear state delayed equation whose coefficient matrices contain norm-bounded time-varying elements. By some matching conditions, we can rend time-varying elements and reform the equation to linear state delayed equation with disturbances. Then we apply a linear transformation technique to reduce the uncertain systems to ones of which nominal systems are of delay-free type. Consequently, we can derive a suitable controller for the perturbed systems, and we will prove that the controller can robustly stabilize the closed-loop systems against perturbation. Finally, control system design for the two tanks chemical reactor with delayed recycle will be illustrated to show applicability of the proposed method.

KEY WORDS – Robust Control, Time-delay System, Control System Design.

1. INTRODUCTION

It is well known that time delay is frequently a source of instability. On the other hand, it is reasonable to include uncertain parameters and disturbance in practical control systems containing modeling errors, linearization approximations, etc. Therefore, the problem of robust stabilization of state delayed systems with uncertain parameters has received considerable attention of many researchers, and many solution approaches have been proposed [1-6].

In this paper we consider a class of time-delay systems containing uncertain parameters and additive disturbances as in [7]. Determination of controller parameters can be devided into two parts. First, the linear transformation propose by Fiagbedzi and Pearson [2] [8] is used to transform the original problem into a equivalent one which is easier to solve. Next, by using the well known Lyapunov min-max approach of Gutman [9], a suitable stabilizing control law is derived in the second part. Finally, An example of product stream control of chemical reactor is given.

2. PROBLEM FORMULATION

Consider a class of uncertain time-delay systems (S_d) which defined by the following state equations

$$\begin{aligned}\dot{x}(t) &= \left[A + \Delta A(t)\right]x(t) + \left[A_h + \Delta A_h(t)\right]x(t-h) \\ &\quad + \left[B + \Delta B(t)\right]u(t) + Bw(t)\end{aligned} \tag{1}$$

where $x \in R^n$ is the current value of the system state, $u(t) \in R^m$ is control function, $w(t) \in R^l$ is the additive disturbance A, A_h, B are known constant matrices of appropriate dimensions, $\Delta A(t)$, $\Delta A_h(t)$, $\Delta B(t)$ are matrices whose elements are continuous, unknown but bounded functions, $h \in R^+$ is a known constant delay time and let initial function of the system be specified as $x_0(\eta) \in C_d\left([-h,0]; R^n\right)$ where C_d denote the Banach space of continuous vector-valued functions defined on an internal $[-h,0]$ taking values in R^n with norm: $\|\varphi\|_d := \sup_{-h\le\eta\le 0}\|\varphi(\eta)\|$ where $\varphi \in C_d\left([-h,0]; R^n\right)$.

We propose a methods of controller design for stabilizing a uncertain time-delay system.

3. Assumptions and Transformation Technique

3.1 Assumptions

Before proposing our controllers, the following assumptions are made throughout here.

3.1.1 Assumption 1: The nominal system of (S_d); i.e., the system (S_d) which $\Delta A(t)=\Delta A_h(t)=0$, $\Delta B(t)=0$, $w(t)=0$ are spectrally stabilizable.

3.1.2 Assumption 2: For all $t \in R^+$, there are exist continuous matrix functions $H(t)$, $H_h(t)$, and $E(t)$ of appropriate dimensions such that

a) $\Delta A(t)=BH(t)$,
b) $\Delta A_h(t)=BH_h(t)$,
c) $\Delta B(t)=BE(t)$,
d) $I+\frac{1}{2}\left(E(t)+E^T(t)\right)\geq \delta I$ for some scalar $\delta > 0$,
e) there are scalar $\mu(x_t)$ and $\mu_E(t)$ such that

$$\mu(x_t)\geq \left\|H(t)x(t)+H_h(t)x(t-h)+w(t)\right\|,$$

and

$$\mu_E(t)\geq \left\|E(t)\right\|$$

where x_t is the restriction function of x to the interval $[t-h,\ t]$ translated to $[-h,\ 0]$; i.e., $x_t \in C_d$ and $x_t(\eta) = x(t+\eta),\ -h\leq \eta \leq 0$

Note that if matching conditions defined in Assumption 2 are satisfies, we can rewrite system (S_d) to the form

$$\dot{x}(t)=Ax(t)+A_h x(t-h)+B(u(t)+v(t)) \tag{2}$$

where

$$v(t)=H(t)x(t)+H_h(t)x(t-h)+E(t)u(t)+w(t) \tag{3}$$

3.2 Transformation Technique

Consider the linear transformation T_c defined by

$$\begin{aligned} z(t) &= \left(T_c(x)\right)(t) \\ &= x(t)+\int_{-h}^{0} e^{A_c\theta}A_h x(t-h-\theta)d\theta \end{aligned} \tag{4}$$

where $A_c \in R^{n\times n}$ is a matrix yet to be defined.

Proposition 3.1: Let the matrix A_c be defined by

$$A_c = A+e^{-hA_c}A_h \tag{5}$$

and

$$\sigma_u(S_d)\subset \sigma(A_c)\subset \sigma(S_d) \tag{6}$$

where

$$\sigma(S_d)=\{s\in C;\ \det(sI-A-e^{-hs}A_h)=0\}$$

and

$$\sigma_u(S_d)=\{s\in \sigma(S_d);\ \mathrm{Re}(s)\geq 0\},$$

Then, $\dot{x}(t)$ satisfies eqn. 1 and hence eqn. 2, if and only if $\dot{z}(t)$ satisfies the system of the form (S_o)

$$\dot{z}(t)=A_c z(t)+B(u(t)+v(t)) \tag{7}$$

Consequently, by this linear transformation, asymptotic stability of $z(t)$ implies asymptotic stability of $x(t)$. Furthermore, the following properties are true:

(a) $(A_c,\ B)$ is a stabilizable pair,
(b) if $\lim_{t\to\infty}\|z(t)\| = 0$, then $\lim_{t\to\infty}\|x(t)\| = 0$
(c) if $\lim_{t\to\infty}\|z(t)\| \leq k_1$, $\exists k_1 < \infty$, then $\lim_{t\to\infty}\|x(t)\| \leq k_2$, $\exists k_2 < \infty$,

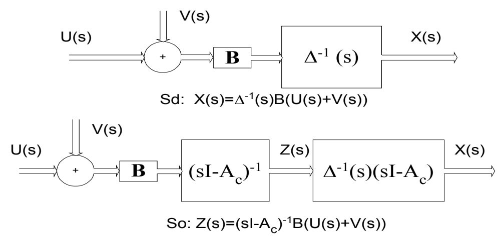

Fig 1. block diagram of (S_d) and (S_o)

Proof:
By using the *Leibniz's formula* [10], it is straightforward to verify that eqn. 2 in conjunction with the transformation eqn. 4 yields eqn. 7; see Appendix (Section 9.1). Property (a) follows from Theorem 3.2 of [8]. To show the property (b) and (c), are obtained using Laplace transform eqn. 4 to obtain, after some rearrangement (see also Fig. 1),

$$X(s) = \Delta^{-1}(s)\left(sI-A_c\right)Z(s)+\Delta^{-1}(s)\left(sI-A_c\right)\Psi(s) \tag{8}$$

where $\Delta(s)=\left\lfloor sI-A-e^{-hs}A_h\right\rfloor$, and

$$\Psi(s) = \int_{-h}^{0} e^{A_c\theta}A_h\int_{-(h+\theta)}^{0} e^{-s(\tau+h+\theta)}x_0(\tau)d\tau d\theta.$$

Next by setting $t = \tau+h+\theta$, Observe that

$$\int_{-(h+\theta)}^{0} e^{-s(\tau+h+\theta)}x_0(\tau)d\tau = \int_{0}^{(h+\theta)} e^{-st}x_0(t-h-\theta)dt$$

Since $x_0(\tau) = 0,\ \forall \tau \notin [-h,\ 0]$, we have

$$\int_{-(h+\theta)}^{0} e^{-s(\tau+h+\theta)} x_0(\tau)d\tau = \int_0^{\infty} e^{-st} x_0(t-h-\theta)dt = L\{x_0(t-h-\theta)\}.$$

This implies that

$$\psi(t) = L^{-1}\{\Psi(s)\} = L^{-1}\left\{\int_{-h}^{0} e^{A_c\theta} A_h L\{x_0(t-h-\theta)\} d\theta\right\} = \int_{-h}^{0} e^{A_c\theta} A_h x_0(t-h-\theta)d\theta$$

and hence,

$$\psi(t) = 0, \ \forall t > h .$$

Note here that eqn. 6 implies that all eigenvalues of the transfer function $\Delta^{-1}(s)(sI - A_c)$ are stable. Consequently, it can be verified that

$$\lim_{t\to\infty}\|x(t)\| \le \lim_{t\to\infty}\left\|L^{-1}\{\Delta^{-1}(s)(sI-A_c)Z(s)\}(t)\right\| + \lim_{t\to\infty}\left\|L^{-1}\{\Delta^{-1}(s)(sI-A_c)\Psi(s)\}(t)\right\| = \lim_{t\to\infty}\left\|L^{-1}\{\Delta^{-1}(s)(sI-A_c)Z(s)\}(t)\right\|.$$

The above analysis imply that $\psi(t)$ does not influence stability of $x(t)$ and it can be verified that stability of $z(t)$ implies asymptotic stability of $x(t)$.

4. CONTROLLER DESIGN

Theorem 4.1 : Suppose there exists a transforma-tion satisfying the hypothesis of proposition 3.1. Then, for given $Q > 0$, there exist a positive definite solution P to the Riccati equation

$$A_c^T P + PA_c - PBB^T P + Q = 0 \tag{9}$$

Furthermore, a stabilizing control law is given by

$$u(t) = u_L(t) + u_N(t) \tag{10}$$

where

$$u_L(t) = -\frac{1}{2}B^T Pz(t) \tag{11}$$

and

$$u_N(t) = -\frac{\rho^2(x_t)B^T Pz(t)}{\delta(\rho(x_t)\|B^T Pz(t)\| + e^{-\phi t})} \tag{12}$$

where the nonlinear gain

$$\rho(x_t) = \mu_E\|u_L(t)\| + \mu(x_t), \tag{13}$$

and $\phi \in R^+$ and δ is the positive scalar defined in Assumption 2-d.

<u>*Proof:*</u>
First, we take the positive definite function

$$V_z(t) = z^T(t)Pz(t) \tag{14}$$

as Lyapunov function candidate for the system (eqn. 7) with control (eqn. 10). Applying with the Riccati equation, the following is obtained of the deriative of V_z

$$:\dot{V}_z = -z^T(t)\left[A_c^T P + PA_c\right]z(t) + 2z^T(t)PB(u(t)+v(t))$$

By using Control law (10), it can be verified that

$$\dot{V}_z(t) \le -z^T(t)Qz(t) + 2e^{-\phi t}$$

then we have

$$\lambda_{\min}(P)\|z(t)\|^2 \le V_z(t) \le \lambda_{\max}(P)\|z(t)\|^2$$

and

$$\dot{V}_z(t) \le -\lambda_{\min}(Q)\|z(t)\|^2 + 2e^{-\phi t}.$$

Next, observe that

$$\dot{V}_z(t) \le -\lambda V_z(t) + 2e^{-\phi t}$$

where

$$\lambda = \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}.$$

Now, let

$$s(t) := \dot{V} + \lambda V_z(t) - 2e^{\phi t}$$

then we have

$$s(t) \le 0$$

and

$$\dot{V}_z(t) = s(t) - \lambda V_z(t) + 2e^{-\phi t}$$

So it can be verified that

$$V_z(t) = V_z(0)e^{-\lambda t} + e^{-\lambda t}\int_0^t e^{\lambda t}\left[s(t) + 2e^{-\phi t}\right]dt \le V_z(0)e^{-\lambda t} + 2e^{-\lambda t}\int_0^t e^{\lambda t}.e^{-\phi t}dt = V_z(0)e^{-\lambda t} + 2e^{-\lambda t}\left[\frac{e^{t(\lambda-\phi)}-1}{(\lambda-\phi)}\right]$$

Consequently,

$$\|z(t)\|^2 \le \frac{V_z(0)e^{-\lambda t}}{\lambda_{\min}(P)} + \frac{2e^{-\lambda t}}{\lambda_{\min}(P)}\left[\frac{e^{t(\lambda-\phi)}-1}{(\lambda-\phi)}\right]$$

The above analysis implies that

$$\lim_{t\to\infty}\|z(t)\|^2 = 0 \tag{15}$$

Since stability of $z(t)$ implies stability of $x(t)$ as shown in proposition 3.1, we can now conclude that closed-loop system is asymptotically stable.

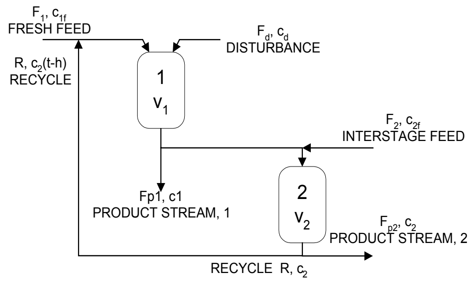

Fig 2. Two stage chemical reactor train with delay recycle

5. ILLUSTRATIVE EXAMPLE

Now we show how to control the two stage chemical reactor with delayed recycle stream, shown in Fig 2. Reactor recycle not only increase the overall conversion, but also reduces the cost of a reaction, therefore, it is very popular in industry. In order to recycle, the input to be recycled must be separated, from the yields, then travel through pipes after separation. This total time of recycle introduced delays in the state.

Consider the irreversible reaction $A \to B$ with negligible heat effect is carried out in the two stage reactor system. Reactor temperature is maintained constant so that only the composition of product streams from the two reactors c_1, c_2 need be controlled. The manipulated variables are the feed compositions to the two reactors, c_{1f}, c_{2f} and the process disturbance is an extra feed stream, F_d whose composition c_d varies because it comes from another processing unit. The flow rates to the reactor system are fixed and only the compositions vary. Suppose, at the input, that the fresh feed of pure A is to be mixed with the recycle stream of unreacted A with recycle flow rate R. Let t be instant of time. Then the material balance equations for the reactor system are

$$V_1\dot{c}_1 = F_1 c_{1f}(t) + Rc_2(t-h) + F_d c_d(t) - (F_1 + R + F_d)c_1(t) - V_1(k_1 + \delta k_1(t))c_1(t) \quad (16)$$

and

$$V_2\dot{c}_2 = (F_1 + R + F_d - F_{p1})c_1(t) + F_2 c_{2f}(t) - (F_{p2} + R)c_2(t) - V_2(k_2 + \delta k_2(t))c_2(t) \quad (17)$$

where the second product stream, F_{p2}, is given by

$$F_{p2} = F_1 + F_d - F_{p1} + F_2$$

Note that the time-varying parameters $\delta k_1(t)$ and $\delta k_2(t)$ represent uncertainties of the system. In practice, exact values of both of parameters are unknown. Nevertheless, it is reasonable to assume that their upper bound values are known; i.e., the information δ_1 and δ_2 such that

$$\delta_1 = \max_t \|\delta k_1(t)\|$$

and

$$\delta_2 = \max_t \|\delta k_2(t)\|$$

respectively, are available. For any given set point (c_{1s}, c_{2s}), our objective is to find a state feedback controller that make c_1 and c_2 converge to c_{1s} and c_{2s} respectively. To achieve this, we define the variables

$$\theta_1 = \frac{V_1}{F_1 + R + F_d}, \quad \theta_2 = \frac{V_2}{F_{p2} + R}$$

$$u_1 = c_{1f} - c_{1fs}, \quad u_2 = c_{2f} - c_{2fs}$$

$$x_1 = c_1 - c_{1s}, \quad x_2 = c_2 - c_{2s}, \quad d = c_d - c_{ds}$$

where c_{ds} is a constant nominal value of the disturbance c_d, and c_{1fs}, c_{2fs} can be obtained from

$$c_{1fs} = -\frac{R_{c2s} + F_d c_{ds} - (F_1 + R + F_d)c_{1s} - V_1 k_1 c_{1s}}{F_1}$$

$$c_{2fs} = -\frac{(F_1 + R + F_d - F_{p1})c_{1s} - (F_{p2} + R)c_{2s} - V_2 k_2 c_{2s}}{F_2}$$

Consequently, the material balance eqn. 15 and 16 can be described by

$$\dot{x}_1(t) = -\left(\frac{1}{\theta_1} + k_1 + \delta k_1(t)\right)x_1(t) + \frac{R}{V_1}x_2(t-h) + \frac{F_1}{V_1}u_1(t) + \frac{F_d}{V_1}d(t) + \delta k_1(t)c_{1s} \quad (18)$$

$$\dot{x}_2(t) = -\left(\frac{1}{\theta_2} + k_2 + \delta k_2(t)\right)x_2(t) + \frac{F_{p2} - F_2 + R}{V_2}x_1(t) + \frac{F_2}{V_2}u_2(t) + \delta k_2(t)c_{2s} \quad (19)$$

Note that $(c_1(t), c_2(t)) \to (c_{1s}, c_{2s})$ whenever $(x_1(t), x_2(t)) \to (0,0)$; therefore, the objective can be achieved by stabilizing the above system described by eqns. 18 and 19. Next, define a state vector

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

It now can be verified that state space description for eqns. 18 and 19 is of the form (S_d) with

$$A = \begin{bmatrix} -(\frac{1}{\theta_1} + k_1) & 0 \\ \frac{F_{P2} - F_2 + R}{V_2} & -(\frac{1}{\theta_2} + k_2) \end{bmatrix}$$

$$\Delta A(t) = \begin{bmatrix} \delta k_1(t) & 0 \\ 0 & \delta k_2(t) \end{bmatrix},$$

$$\|\delta k_1(t)\| \le \delta_1, \ \|\delta k_2(t)\| \le \delta_2,$$

$$A_h = \begin{bmatrix} 0 & \frac{R}{V_1} \\ 0 & 0 \end{bmatrix}, \ \Delta A_h(t) = 0,$$

$$B = \begin{bmatrix} \frac{F_1}{V_1} & 0 \\ 0 & \frac{F_2}{V_2} \end{bmatrix}, \ \Delta B(t) = 0,$$

$$w(t) = \begin{bmatrix} \frac{F_d d(t)}{F_1} \\ 0 \end{bmatrix}, \ \|\delta k_1(t)\| \le d_{\max}.$$

To illustrate the proposed controller design, let us choose

$$k_1 = k_2 = 1, \ v_1 = v_2 = 1,$$

$$F_1 = 0.4, \ F_2 = 0.5,$$

$$F_{p_1} = 0.5, \ F_{p_2} = 0.5,$$

$$F_d = 0.1, \ R = 0.25, \ h = 1,$$

$$\delta_1 = 0.4, \ \delta_2 = 0.5, \ \delta_3 = 0.5,$$

so that

$$\theta_1 = 0.75, \ \theta_2 = 0.5,$$

and hence

$$A = \begin{bmatrix} -1.75 & 0 \\ 0.25 & -1.75 \end{bmatrix}, \ A_h = \begin{bmatrix} 0 & 0.25 \\ 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$H(t) = \begin{bmatrix} \frac{\delta k_1(t)}{0.4} & 0 \\ 0 & \frac{\delta k_2(t)}{0.5} \end{bmatrix}, \ \|H(t)\| \le 1,$$

$$w(t) = \begin{bmatrix} \frac{d(t)}{4} \\ 0 \end{bmatrix}, \ \|w(t)\| \le 0.125$$

Note here that the nominal system is stable. Indeed, it can be verified that $s_1 = -2.72791$ and $s_2 = -1.27667$ are the poles of the nominal system. Based on the procedure given in [8] with $\sigma(A_c) = \{s_1, s_1\}$ the required matrix parameter A_c of the transformation is then determined to be

$$A_c = \begin{bmatrix} -1.75 & 1.851497 \\ 0.25 & -2.254575 \end{bmatrix}$$

Next, solve to Lyapunov equation (2.3.12) with $Q = I$ to get

$$P = \begin{bmatrix} 0.3093344 & 0.165341 \\ 0.165341 & 0.357552 \end{bmatrix}$$

A suitable control law is then given by eqn. 9 with $\delta = 1$, $\phi = 0.5$ and

$$\rho(x_t) = \sqrt{(x_1(t) + c_{1s})^2 + (x_2(t) + c_{2s})^2} + 0.125$$

Suppose that set point is chosen as

$$c_{1s} = 0.5, \ c_{2s} = 1.0$$

Simulations are now presented for the corresponding closed-loop system. In these simulations, the uncertain parameters are taken to be as follows.

$$\delta k_1(t) = 0.4\sin(2t), \ \delta k_2(t) = 0.5\sin(2t),$$

$$d(t) = 0.5\sin(2t).$$

The initial condition is taken to be $x_0 = [-0.4 \quad -1.0]^T$ on [-1, 0]. The results of these simulations are shown in Fig 3.

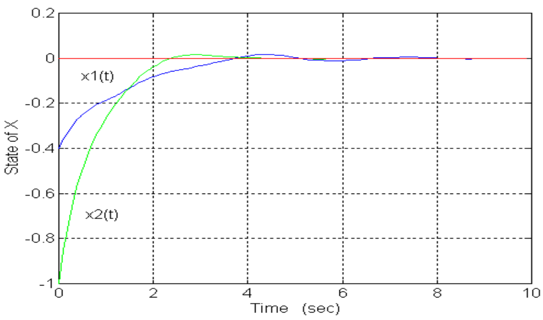

Fig 3. Response of state x versus time

6. CONCLUSION

We have presented a computational method to stabilize uncertain systems including known constant time delay. By using the matching conditions, we can change system model (eqn. 1) into new model (eqn. 2) that is easier for analysis. We then use a linear transformation [2] to reduce the delay system model with A_c, which have been chosen so that $\Delta^{-1}(s)(sI - A_c)$ stable. This explains why stability of $z(t)$ can imply stability of $x(t)$

In comparison with [7], the advantage we presented is the control law (10); by changing constant ε to $e^{-\phi t}$ that converges to zero. It therefore controls the system more efficiently with better performance. Finally, we show how to apply the proposed stabilization method to set point control of a chemical reactor train with delay recycle.

7. ACKNOWLEDGMENT

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9. APPENDIX

9.1 *Supplementary proof of Proposition 3.1*

From the hypothesis of Proposition 3.1, we have

$$\dot{x}(t) = Ax(t) + A_h x(t-h) + B(u(t) + v(t)) \tag{20}$$

with the auxilary output

$$\begin{aligned} z(t) &= (T_c(x))(t) \\ &= x(t) + \int_{-h}^{0} e^{A_c\theta} A_h x(t-h-\theta)\,d\theta \end{aligned} \tag{21}$$

where the matrix A_c be defined by

$$A_c = A + e^{-hA_c} A_h \tag{22}$$

By using the *Leibniz's formula* [10], it can be verified that

$$\begin{aligned} \frac{d}{dt}\int_{-h}^{0} e^{A_c\theta} A_h x(t-h-\theta)\,d\theta \\ &= e^{-hA_c} A_h x(t) - A_h x(t-h-\theta) \\ &+ A_c \int_{-h}^{0} e^{A_c\theta} A_h x(t-h-\theta)\,d\theta \end{aligned}$$

Hence,

$$\begin{aligned} \dot{z}(t) &= \dot{x}(t) + \frac{d}{dt}\int_{-h}^{0} e^{A_c\theta} A_h x(t-h-\theta)\,d\theta \\ &= Ax(t) + A_h x(t-h) + B(u(t)+v(t)) \\ &+ e^{-hA_c} A_h x(t) - A_h x(t-h-\theta) \\ &+ A_c \int_{-h}^{0} e^{A_c\theta} A_h x(t-h-\theta)\,d\theta \\ &= A_c\left[x(t) + \int_{-h}^{0} e^{A_c\theta} A_h x(t-h-\theta)\,d\theta\right] \\ &+ B(u(t)+v(t)) + \left[A + e^{-hA_c} A_h - A_c\right] x(t) \end{aligned}$$

which is equivalent to

$$\dot{z}(t) = A_c z(t) + B(u(t)+v(t))$$

as in eqns. 21 and 22

Next, to show eqn. 8, Laplace transform eqn. 21 to obtain

$$\begin{aligned} Z(s) &= L\{z(t)\} \\ &= L\left\{x(t) + \int_{-h}^{0} e^{A_c\theta} A_h x(t-h-\theta)\,d\theta\right\} \\ &= X(s) + \int_{-h}^{0} e^{A_c\theta} A_h L\{x(t-h-\theta)\}\,d\theta \end{aligned}$$

Since, for any scalar $\alpha > 0$,

$$\begin{aligned} L\{x(t-\alpha)\} &= \int_{0}^{\infty} e^{-st} x(t-\alpha)\,dt \\ &= e^{-s\alpha} X(s) + \int_{-\alpha}^{0} e^{-s(\tau+\alpha)} x_0(t-\tau)\,d\tau \end{aligned}$$

where $x_0 \in C_d\left([-\alpha,\ 0];\ R^n\right)$ denote the initial function. Consequently,

$$\begin{aligned} Z(s) &= X(s) + \int_{-h}^{0} e^{A_c\theta} A_h e^{-s(h+\theta)}\,d\theta X(s) \\ &+ \int_{-h}^{0} e^{A_c\theta} A_h \int_{-(h+\theta)}^{0} e^{-s(\tau+h+\theta)} x_0(\tau)\,d\tau\,d\theta \\ &= \left[I + \int_{-h}^{0} e^{A_c\theta} A_h e^{-s(h+\theta)}\,d\theta\right] X(s) \\ &+ \int_{-h}^{0} e^{A_c\theta} A_h \int_{-(h+\theta)}^{0} e^{-s(\tau+h+\theta)} x_0(\tau)\,d\tau\,d\theta \end{aligned} \tag{23}$$

Note here that

$$I + \int_{-h}^{0} e^{A_c\theta} A_h e^{-s(h+\theta)}\,d\theta = \left(sI - A_c\right)^{-1} \Delta(s) \tag{24}$$

where $\Delta(s) = \left|sI - A - e^{-hs} A_h\right|$. This can be verified easily by direct integration and then using eqn 22. Finally, direct substitution of eqn. 24 in eqn. 23 yields the required result.

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