Development of Grid Generation Software

Pitiya Namwong and Varangrat Juntasaro
CML, Department of Mechanical Engineering, Faculty of Engineering,
Kasetsart University, Bangkok, Thailand.

ABSTRACT - Grid generation is the process of determining the grid point distribution on the considered boundary. Each grid point represents interested properties; consequently, this process is the most important step in computational mechanics as it affects directly to the precision of the calculation of exact properties. This work is a software development for structured grid generation using numerical method of elliptic partial differential equation. The software can support both two and three dimensional spaces. Functions of the program can be categorized into 3 major parts. The first part is the object's boundary defining by AutoCAD. The second part is the grid point generation within boundary by using the developed program designed by C language on Visual C++. And the last part is to graphical demonstrate the results using the visualization software called SCV developed by parallel research group of computer engineering department, Kasetsart University. This developed software is successfully designed for practical two and three dimensional spaces with the user-friendly interface linking the input to the software's calculation. The interface is also capable of linking the boundary information from AutoCAD software into the calculation part. This is extremely useful for complicated boundary object. Several samples of different boundary were tangibly demonstrated including rectangular boundary, curved boundary, backward facing step boundary, airfoil boundary, car boundary, and free-form-customized boundary from AutoCAD.

KEY WORDS - Grid Generation, Numerical Method, Software Development

บทคัดย่อ – กระบวนการสร้างกริด คือ กระบวนการสร้างจุดกริดภายในขอบเขตที่ต้องการศึกษา โดยจุดกริดแต่ละจุดจะแทนค่าคุณสมบัติของตัวแปรที่ต้องการศึกษา ดังนั้นกระบวนการสร้างกริดจึงเป็นขั้นตอนที่สำคัญที่สุดในวิธีการทางกลศาสตร์เชิงคำนวณเพราะการกระจายของจุดกริดมีผลโดยตรงกับความถูกต้องแม่นยำในการคำนวณหารูปแบบที่แท้จริงของตัวแปรที่กำลังศึกษาอยู่ งานวิจัยนี้เป็นการพัฒนาโปรแกรมคอมพิวเตอร์สำหรับกระบวนการสร้างกริดแบบมีโครงสร้างโดยอาศัยหลักการของระเบียบวิธีทางตัวเลขจากสมการเชิงอนุพันธ์ย่อยแบบเอลลิปติกในการคำนวณการกระจายจุดกริด ซอฟต์แวร์ที่พัฒนาขึ้นนี้แบ่งการใช้งานออกเป็น 2 ส่วนคือ กระบวนการสร้างกริดแบบมีโครงสร้างในสองมิติ และกระบวนการสร้างกริดแบบมีโครงสร้างในสามมิติ โดยในการออกแบบโปรแกรมกระบวนการสร้างกริดสำหรับงานวิจัยนี้ แบ่งขั้นตอนการใช้งานเป็น 3 ขั้นตอนคือขั้นตอนแรกเป็นการนำโปรแกรมออโตแคดเข้ามาช่วยในการกำหนดรูปแบบและขอบเขตเริ่มต้น ขั้นตอนที่ 2 คือการเขียนภาษาซีบนโปรแกรมวิชวลซี พลัส พลัส ในการคำนวณการกระจายจุดกริดภายใน ส่วนขั้นตอนสุดท้าย คือ การแสดงผลที่ได้ในรูปกราฟฟิกซึ่งทำให้ง่ายต่อการวิเคราะห์ผลลัพธ์ที่ได้ โปรแกรมกระบวนการสร้างกริดที่พัฒนาขึ้นนี้ถูกออกแบบให้ง่ายต่อการใช้งานทั้งในสองมิติและสามมิติ โดยมีหน้าต่างช่วยในการรับส่งข้อมูลที่ป้อนจากผู้ใช้เข้าสู่โปรแกรมคำนวณ รวมถึงสามารถกำหนดขอบเขตที่ต้องการจากโปรแกรมออโตแคดซึ่งเป็นผลดีต่อการพัฒนากระบวนการสร้างกริดบริเวณขอบเขตที่ซับซ้อนในอนาคต

คำสำคัญ – กระบวนการสร้างกริด, ระเบียบวิธีเชิงตัวเลข, การพัฒนาโปรแกรม

1. Introduction

Grid Generation is the process of grid assignment within the interested boundary. Each grid point represents the interested parameter such as flow, speed, pressure or temperature. The grid generation is the most important step in computational mechanics since the grid distribution directly affects to the accuracy of calculation of the interested parameter. For example, in the case of fluid flow study, in the area that has more change in flow, the density of grid point should be increased to identify the change more precisely. Therefore, computational grid generation is required for timely and accurate grid point distribution. The principal of grid generation is to start with the step of defining the boundary. Then using the numerical method of grid generation, the grid point distribution within the assigned boundary is calculated. A different program then graphically demonstrates the result. In this study, the first step of boundary defining use AutoCAD software. The second step is grid distribution calculation using the numerical method of elliptic partial differential equation and then the third step is graphical demonstration by SCV (Scientific Computational Visualization) program. The program is also designed to be user-friendly.

The developed program does not only alleviate the high cost foreign software but also is designed to be a prototype of the grid generation software for the two and three dimensional structured grid generation for future development of grid generation software.

2. Numerical method and related theory of structured grid generation for two and three-dimensional spaces

Structured grid generation involves the coordinate transformation that transforms the body-fitted nonuniform nonorthogonal physical space xyz into the uniform orthogonal computational space $\xi\eta\phi$, rectangular space. Furthermore, through this transformation, the governing partial differential equations must be utilized in order to apply in this transformed, rectangular grid [5]. Three classifications of partial differential equation (i.e., elliptic, parabolic, or hyperbolic) can be used as the governing grid generation differential equation. For elliptic equations, there are no limited regions of domain and information is propagated everywhere in all direction. This is in contrast to the marching solutions germane to parabolic and hyperbolic equations. For this reason, problem involving elliptic equations are suggested to be the governing partial differential equations, because the solution within the domain depends on the total boundary domain, boundary conditions must be applied over the entire boundary [1]. The governing partial differential equations are solved by finite difference method carried out in the computational space. There are two basic steps in governing elliptic partial differential equations: specification of the boundary point distribution and determination of the interior point distribution [1]. Moreover, the accuracy of the solution depends on correlation between the number of grid point and the magnitude of gradients. The large gradients require more grid point to be clustered. In the other hand, small gradients require grid points to be spread out in the region.

For example, consider an airfoil in rectangular grid, the grid distribution in Fig.1 demonstrates some weak points

1. The airfoil boundary of the physical space does not fall on a coordinate line. Therefore, the application of boundary conditions is difficult.

2. Some points that fall inside the airfoil boundary are not needed for the calculation.

3. More grid points are needed in the area that has large gradient to represent the change of properties.

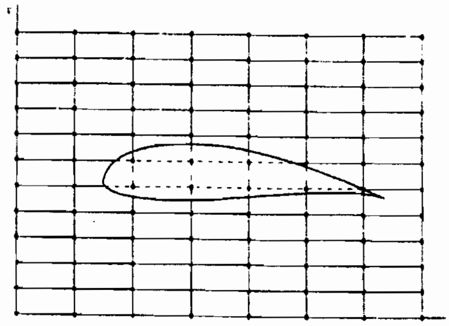

Figure 1. An airfoil in a rectangular grid [1].

By the above reasons, the grid distribution in Fig.1 is not appropriate to the calculation. In the other hand, the more appropriate grid distribution is shown in Fig. 2.

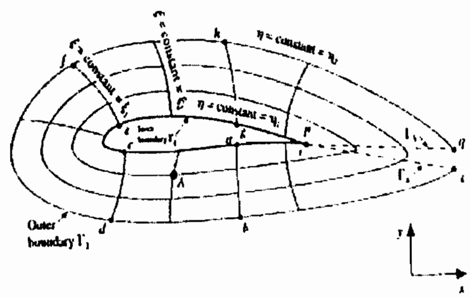

Figure 2. Boundary – fitted grid on physical plane [1].

From the Fig.2, the boundary-fitted coordinate system is shown. The grid distribution of system naturally fall on the airfoil surface but the grid is not rectangular and is nonuniform grid. Consequently, the equation of partial differential equations is difficult. With this reason, the next step is to one-by-one transform the curved grid in physical space to a rectangular grid in terms of ξ and η as the new independent variables. To understand the above grid system, consider the airfoil in Fig.2.

Consider the boundary of the airfoil given in Fig.2. A curvilinear system surrounds the airfoil where one coordinate line $\eta = \eta_1$ = constant which is represented by Γ_1. While the outer boundary is represented by Γ_2 which $\eta = \eta_2$ = constant. Each line which fan out from the inner boundary Γ_1 and which intersect the outer boundary Γ_2 has an equation ξ = constant for example the line "ef" has $\xi = \xi_1$ = constant = 0.1 and the line "gh" may has $\xi = 0.2$ or the line "pq" may has $\xi = 0.3$. Likewise, assign two lines at the end of airfoil: "pq" and "rs". "pq" is Γ_3 boundary and "rs" is Γ_4 boundary. Γ_3 and Γ_4 are represented by $\xi = \xi_3$ and $\xi = \xi_4$ respectively. Therefore Γ_1, Γ_2, Γ_3 and Γ_4 are 4 side of the airfoil boundary. Next step is to transform the nonuniform nonorthogonal physical space illustrated in Fig.2 into the uniform orthogonal computational space illustrated in Fig.3.

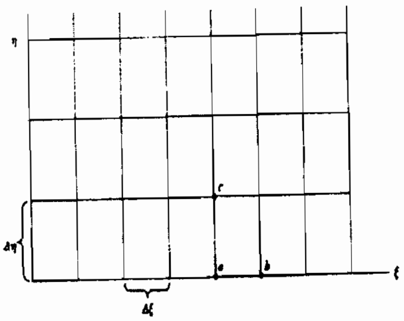

Figure 3. Uniform grid on computational plane [1].

The solving step of grid generation can be done through the defining of the boundary and then calculation of the grid point distribution towards desired direction within the boundary.

Start considering the calculation of two dimensions from below equation. The most common elliptic partial differential equation used for grid generation is the Poisson equation [1]:

$$\nabla^2\xi = \xi_{xx} + \xi_{yy} = P(\xi, \eta)$$

$$\nabla^2\eta = \eta_{xx} + \eta_{yy} = Q(\xi, \eta) \qquad (1)$$

From the chain rule for partial derivatives of the generic function f(x , y) [2].

$$f_x = \xi_x f_\xi + \eta_x f_\eta$$

$$f_y = \xi_y f_\xi + \eta_x f_\eta \qquad (2)$$

Let f = x and make some arrangement, Eq. (1) becomes

$$\alpha x_{\xi\xi} + 2\beta x_{\xi\eta} + \gamma x_{\eta\eta} = -I^2(P x_\xi + Q x_\eta) \qquad (3)$$

In the same manner, for f = y, Eq. (1) becomes

$$\alpha y_{\xi\xi} + 2\beta y_{\xi\eta} + \gamma y_{\eta\eta} = -I^2(P y_\xi + Q y_\eta) \qquad (4)$$

where

$$\alpha = x_\eta^2 - y_\eta^2 \qquad (5)$$

$$\beta = x_\xi x_\eta - y_\xi y_\eta \qquad (6)$$

$$\gamma = x_\xi^2 - y_\xi^2 \qquad (7)$$

The Jacobian determination in two dimension space of the inverse transformation, denoted by I:

$$I = \frac{\partial(x,y)}{\partial(\xi,\eta)} = \begin{vmatrix} x_\xi & x_\eta \\ y_\xi & y_\eta \end{vmatrix}$$

$$I = x_\xi y_\eta - x_\eta y_\xi \qquad (8)$$

Discretizing Eq. (3) and (4) in finite difference from using second-order centered difference approximations of the exact partial derivatives [4].

When the grid point assignment is done on the boundary, then the control of the internal grid distribution is affected by P(ξ,η) and Q(ξ,η). The P value affects the internal point trend to be left or right inclination and the Q value affects the internal point trend to be upward or downward. However, the initial value of the P and Q need to be assumed at the first step and then determine iteratively. So, the P and Q definition in the iteration is shown in the following equation [3].

$$P^{n+1} = P^{n+1} + \Delta P \qquad (9)$$

$$Q^{n+1} = Q^{n+1} + \Delta Q \qquad (10)$$

where n denotes the iteration level. The value of calculation of ΔQ^n is shown below.

$$\Delta Q^n = +\tan^{-1}\left(\frac{\Delta s^n - \Delta s^*}{\Delta s^*}\right) \quad (11)$$

Where Δs^* is the desired spacing.

And the value of calculation of ΔP^n is shown below.

$$\Delta P^n = +\tan^{-1}\left(\frac{\alpha^n - \alpha^*}{\alpha^*}\right) \quad (12)$$

where α^* is the desired angle of intersection.

Similarly, the main equations for the grid point calculation in three dimension are

$$\nabla^2\xi = \xi_{xx} + \xi_{yy} + \xi_{zz} = P(\xi, \eta, \phi)$$

$$\nabla^2\eta = \eta_{xx} + \eta_{yy} + \eta_{zz} = Q(\xi, \eta, \phi)$$

$$\nabla^2\phi = \phi_{xx} + \phi_{yy} + \phi_{zz} = R(\xi, \eta, \phi) \quad (13)$$

From the chain rule for partial derivatives of the generic function f(x , y , z) [2].

$$f_x = \xi_x f_\xi + \eta_x f_\eta + \phi_x f_\phi$$

$$f_y = \xi_y f_\xi + \eta_y f_\eta + \phi_y f_\phi$$

$$f_z = \xi_z f_\xi + \eta_z f_\eta + \phi_z f_\phi \quad (14)$$

Let $f = x$ and make some arrangement, Eq. (13) becomes

$$\alpha_{11} x_{\xi\xi} + 2\alpha_{12} x_{\xi\eta} + 2\alpha_{13} x_{\xi\phi} + \alpha_{22} x_{\eta\eta} +$$

$$2\alpha_{23} x_{\phi\eta} + \alpha_{33} x_{\phi\phi} = -I^2(Px_\xi + Qx_\eta + Rx_\phi) \quad (15)$$

In the same manner, for $f = y$, Eq. (13) becomes

$$\alpha_{11} y_{\xi\xi} + 2\alpha_{12} y_{\xi\eta} + 2\alpha_{13} y_{\xi\phi} + \alpha_{22} y_{\eta\eta} +$$

$$2\alpha_{23} y_{\phi\eta} + \alpha_{33} y_{\phi\phi} = -I^2(Py_\xi + Qy_\eta + Ry_\phi) \quad (15)$$

In the same manner, for $f = z$, Eq. (13) becomes

$$\alpha_{11} z_{\xi\xi} + 2\alpha_{12} z_{\xi\eta} + 2\alpha_{13} z_{\xi\phi} + \alpha_{22} z_{\eta\eta} +$$

$$2\alpha_{23} z_{\phi\eta} + \alpha_{33} z_{\phi\phi} = -I^2(Pz_\xi + Qz_\eta + Rz_\phi) \quad (16)$$

where $f_x = \xi_x f_\xi + \eta_x f_\eta + \phi_x f_\phi$

$$\beta_{11} = I\xi_x = y_\eta z_\phi = y_\phi z_\eta \quad (17)$$

$$\beta_{12} = I\eta_x = y_\phi z_\xi = y_\xi z_\phi \quad (18)$$

$$\beta_{13} = I\phi_x = y_\xi z_\eta = y_\eta z_\xi \quad (19)$$

$$\beta_{21} = I\xi_y = x_\phi z_\eta = x_\eta z_\phi \quad (20)$$

$$\beta_{22} = I\eta_y = x_\xi z_\phi = x_\phi zv \quad (21)$$

$$\beta_{23} = I\phi_y = x_\eta z_\xi = x_\xi z_\eta \quad (22)$$

$$\beta_{31} = I\xi_z = x_\eta y_\phi = x_\phi y_\eta \quad (23)$$

$$\beta_{32} = I\eta_z = x_\phi y_\xi = x_\xi y_\phi \quad (24)$$

$$\beta_{33} = I\phi_z = x_\xi y_\eta = x_\eta y_\xi \quad (25)$$

$$\alpha_{11} = \beta_{11}^2 + \beta_{21}^2 + \beta_{31}^2 \quad (26)$$

$$\alpha_{12} = \beta_{11}\beta_{12} + \beta_{21}\beta_{22} + \beta_{31}\beta_{32} \quad (27)$$

$$\alpha_{13} = \beta_{11}\beta_{13} + \beta_{21}\beta_{23} + \beta_{31}\beta_{33} \quad (28)$$

$$\alpha_{22} = \beta_{12}^2 + \beta_{22}^2 + \beta_{32}^2 \quad (29)$$

$$\alpha_{23} = \beta_{12}\beta_{13} + \beta_{22}\beta_{23} + \beta_{32}\beta_{33} \quad (30)$$

$$\alpha_{33} = \beta_{13}^2 + \beta_{23}^2 + \beta_{33}^2 \quad (31)$$

The Jacobian determination in three dimensions of the inverse transformation, denoted by I:

$$I = \frac{\partial(x,y,z)}{\partial(\xi,\eta,\phi)} = \begin{vmatrix} x_\xi & x_\eta & x_\phi \\ y_\xi & y_\eta & y_\phi \\ z_\xi & z_\eta & z_\phi \end{vmatrix}$$

$$I = x_\xi (y_\eta z_\phi - y_\phi z_\eta) - x_\eta (y_\xi z_\phi - y_\phi z_\xi) +$$

$$x_\phi (y_\xi z_\eta - y_\eta z_\xi) \quad (32)$$

Discretizing Eq. (14), (15) and (16) in finite difference from using second-order centered difference approximations of the exact partial derivatives [4].

When the grid point assignment is done on the boundary, then the control of the internal grid distribution is affected by $P(\xi,\eta,\phi)$, $Q(\xi,\eta,\phi)$ and $R(\xi,\eta,\phi)$. The P value affects the internal point trend to be left or right inclination, the Q value affects the internal point trend to be upward or downward and the R value affects the internal point trend to be forward or backward. However, the initial value of the P, Q and R need to be assumed at the first step and then determine iteratively. So, the P, Q and R definition in the iteration is shown in the following equation [3].

$$P^{n+1} = P^{n+1} + \Delta P^{n} \quad (33)$$

$$Q^{n+1} = Q^{n+1} + \Delta Q^{n} \quad (34)$$

$$R^{n+1} = R^{n+1} + \Delta R^{n} \quad (35)$$

where n denotes the iteration level. The value of calculation of ΔQ^n is shown below.

$$\Delta Q^{n} = +\tan^{-1}\left(\frac{\Delta s^{n} - \Delta s^{*}}{\Delta s^{*}}\right) \quad (36)$$

Where Δs^* is the desired spacing.

And the value of calculation of ΔP^n is shown below.

$$\Delta P^{n} = +\tan^{-1}\left(\frac{\alpha^{n} - \alpha^{*}}{\alpha^{*}}\right) \quad (37)$$

In the same manner, ΔR^n can be calculated as follow.

$$\Delta R^{n} = +\tan^{-1}\left(\frac{\alpha^{n} - \alpha^{*}}{\alpha^{*}}\right) \quad (38)$$

where α^* is the desired angle of intersection.

3. Scope of Work

This work is in accordance with software development for structured grid generation using elliptic partial differential equation as said earlier. Grid generation software is designed by C language on Visual C++ with the user – friendly interface and its steps are divided into three parts. The first step is to specify the boundary point distributions and determine the initial point distribution within boundary by using AutoCAD linked to Visual C++ or directly defining the distributions on Visual C++. For the second step, the interior point distribution is determined by using developed program deriving from elliptic partial differential equation with C language. And the last step is to show the results using the visualization software called SCV developed by parallel research group of computer engineering department, Kasetsart university both two and three dimensions.

4. Example of the program testing in two dimensional space

After executing grid generation software, the first window is shown and selected "2 DIMENSION"

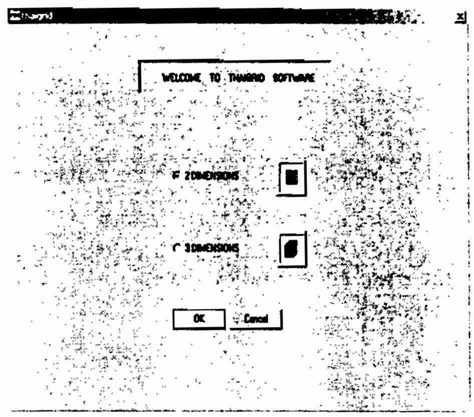

Figure 4. First window of grid generation software for choosing two dimensional calculation.

After the second window prompts, there are 6 boundaries that can be calculated: rectangular boundary, curved boundary, backward facing step boundary, airfoil boundary, car boundary and the boundary from AutoCAD.

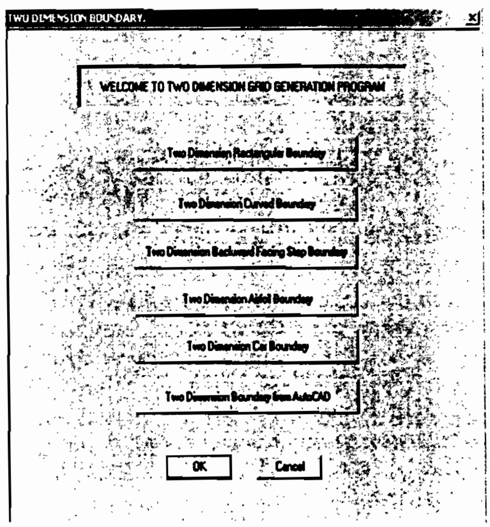

Figure 5. The window shows the boundaries, which can be calculated for 2 dimension.

If the boundary from AutoCAD is selected, the following window will show afterward.

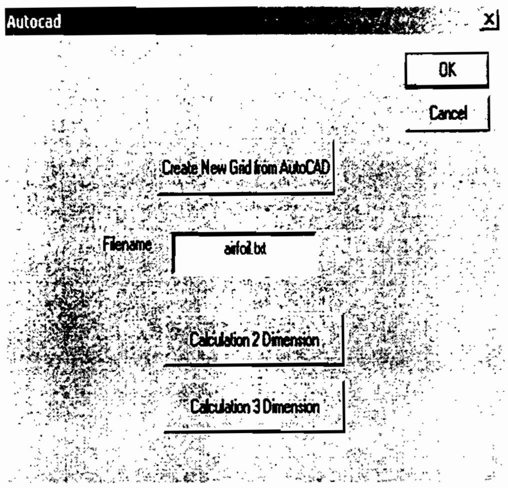

Figure 6. The window for the boundary from AutoCAD.

After choosing "Create New Grid Generation From Autcad" , the program will lead to AutoCAD program. The boundary is drawn. In this case, airfoil boundary from AutoCAD design is used for example shown in Fig.7.

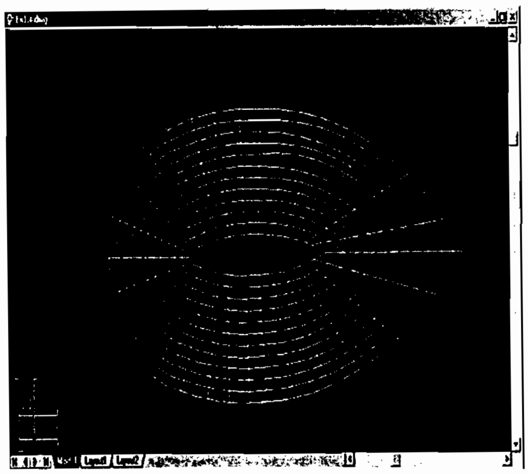

Figure 7. Airfoil boundary is designed on AutoCAD program.

When the control of the grid distribution provides the appropriate distribution, the result shows in Fig.9.
Other results are shown in Fig.10, 11 and 12.

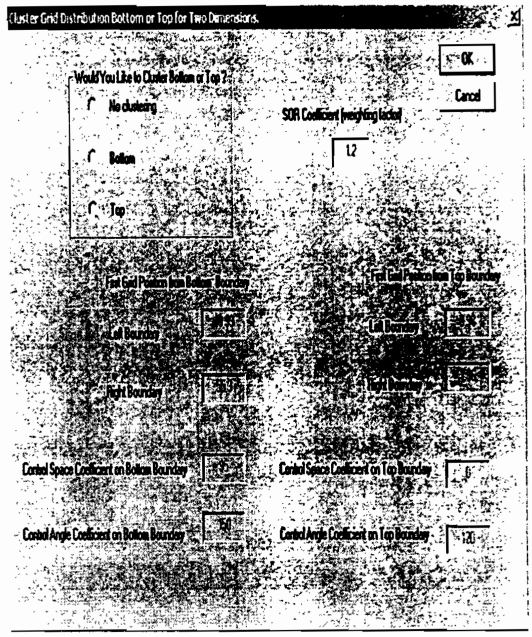

Figure 8. The control value input of the grid distribution for two dimension.

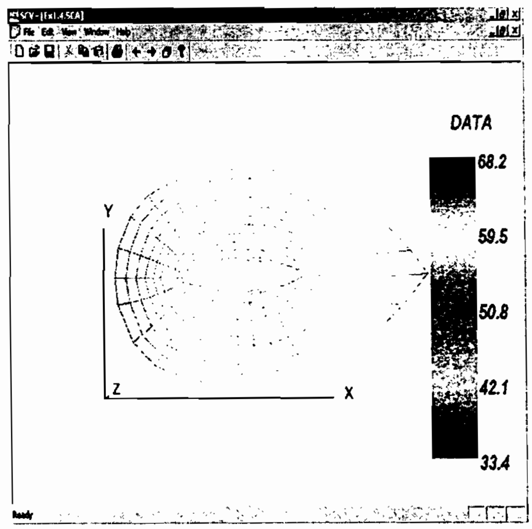

Figure 9. The result of grid distribution within airfoil boundary by using grid within software in two dimension.

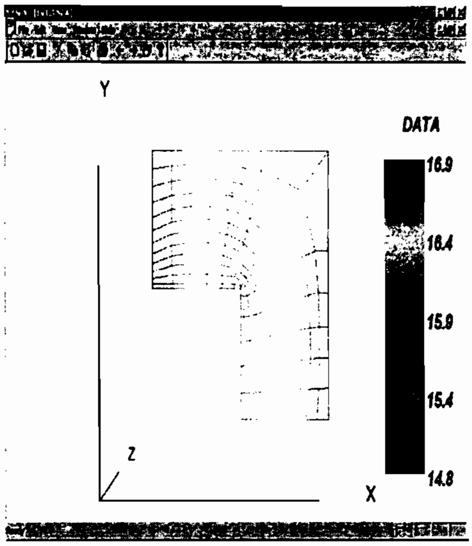

Figure 10 .The result of grid distribution within curved boundary by using grid generation software in two dimension.

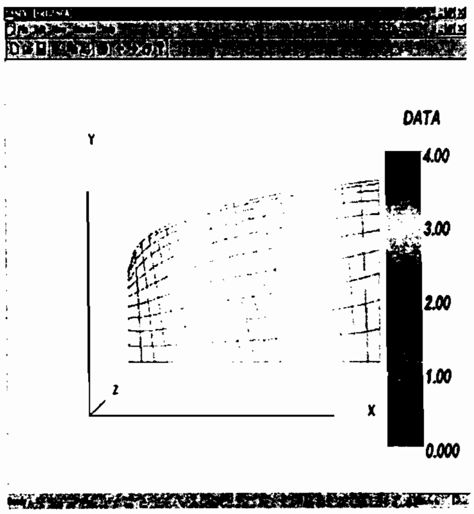

Figure 11. The result of grid distribution within backward facing step boundary by using grid generation software in two dimension.

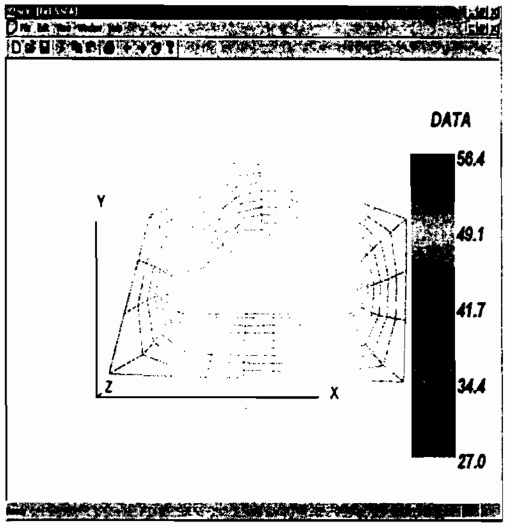

Figure 12. The result of grid distribution within car boundary by using grid generation software in two dimension.

5. Example of program testing in three dimensional space.

After executing grid generation software, the first window is shown and selected "3 DIMENSION"

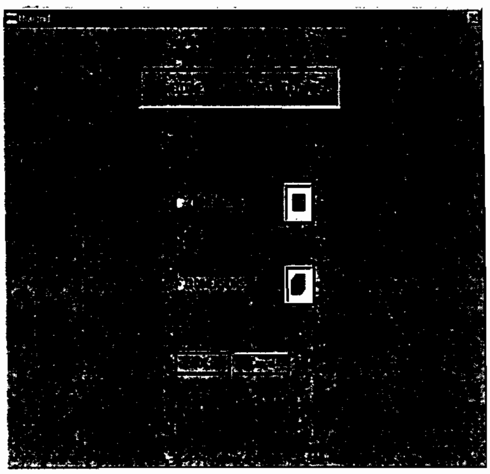

Figure 13. First window of grid generation software for choosing three dimension calculation.

After the second window prompts, there are 6 boundaries that can be calculated: rectangular boundary, curved boundary, backward facing step boundary, airfoil boundary, car boundary and the boundary from AutoCAD.

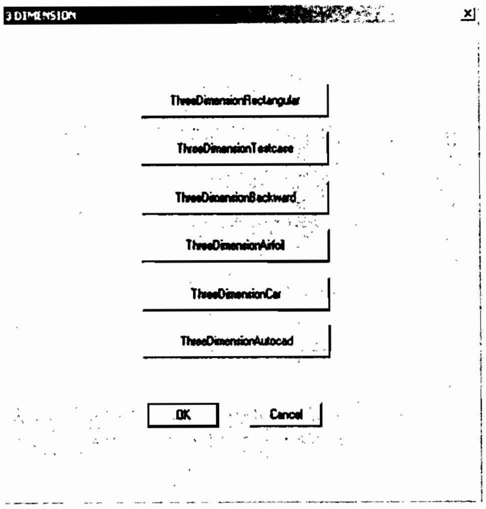

Figure 14. The window shows the boundaries, which can be calculated for three dimension.

If the boundary from AutoCAD is selected, the following window will show afterward.

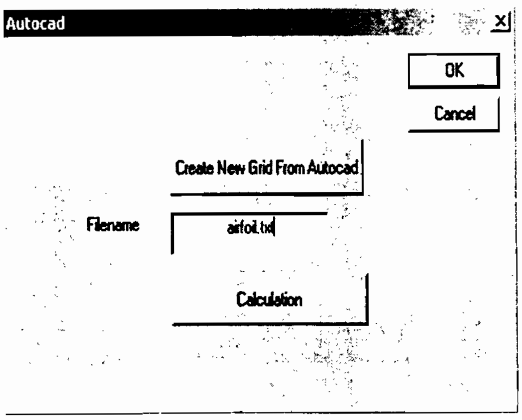

Figure 15. the window for the boundary from AutoCAD.

After choosing "Create New Grid Generation From Autcad" , the program will lead to AutoCAD program. The boundary is then drawn. In this case, airfoil boundary from AutoCAD design is used for example shown in Fig.16.

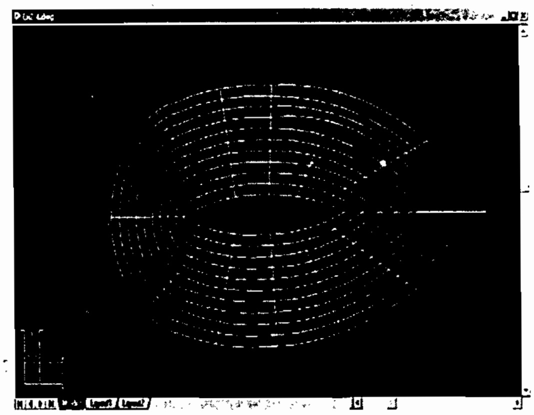

Figure 16. Airfoil boundary is designed on AutoCAD program..

When the control of the grid distribution provide the appropriate distribution, the result shows in Fig.18.
Other results are shown in Fig.19, 20 and 21.

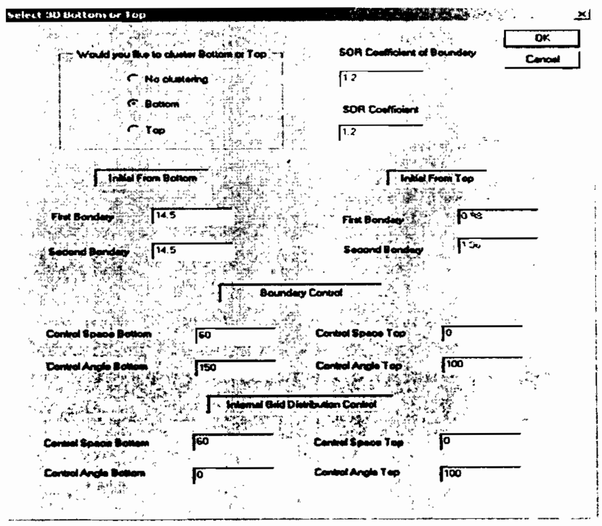

Figure 17. The control value of the grid distribution for three dimension.

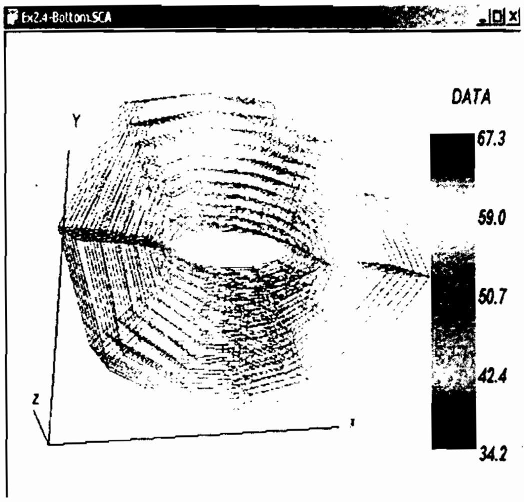

Figure 18. The result of grid distribution within airfoil boundary by using grid generation software in three dimension.

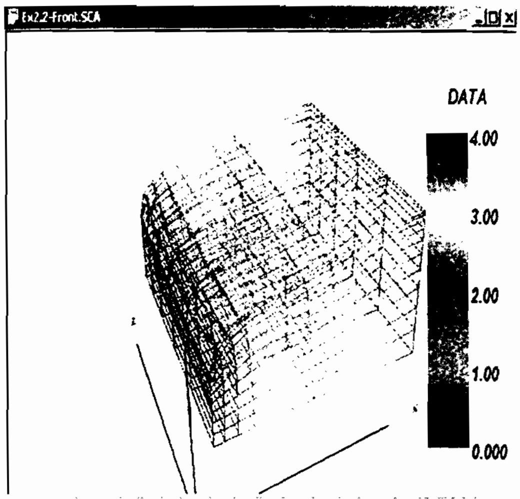

Figure 20. The result of grid distribution within curved boundary by using grid generation software in three dimension.

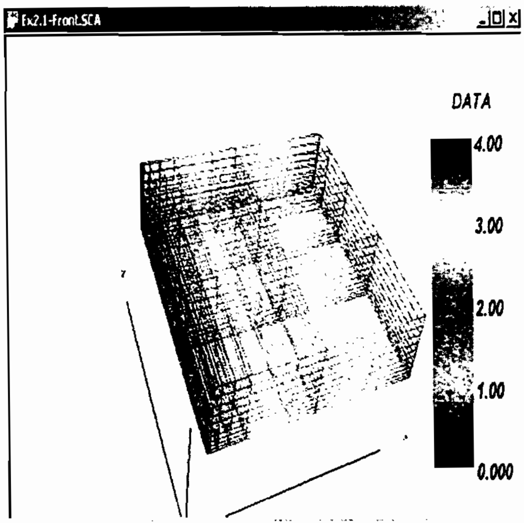

Figure 19. The result of grid distribution within rectangular boundary by using grid generation software in three dimension.

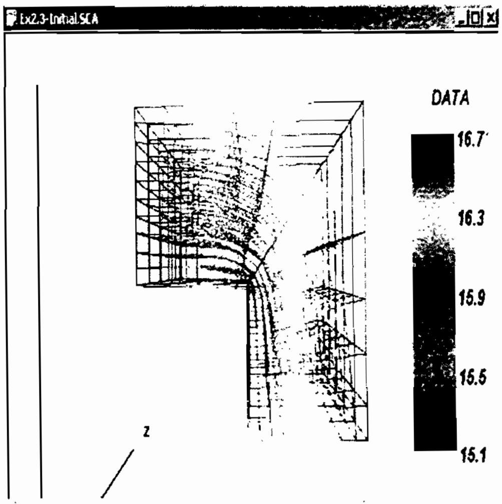

Figure 21. The result of grid distribution within backward facing step boundary by using grid generation software in three dimension.

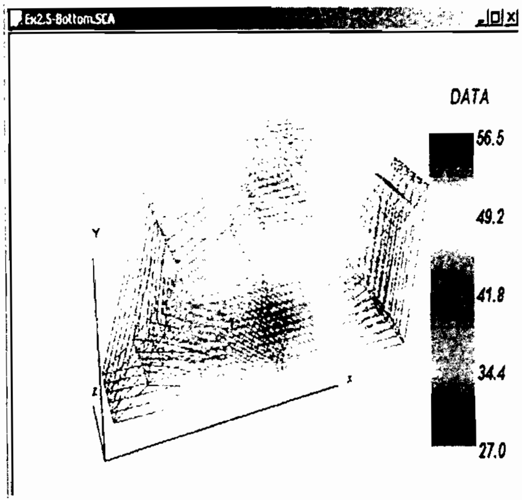

Figure 22. The result of grid distribution within car boundary by using grid generation software in three dimension.

6. Conclusion

This developed software for grid generation is capable of handling two and three dimensional boundaries. With user-friendly interface, users can utilize this software easily. In addition, the boundary defining is simplified by using AutoCAD, which is a foundation for future grid generation study on more complicated boundary. With graphical model, it is easy to investigate the results. Moreover, the result model is in beautiful form and can be scaled down or enlarged as well as rotated by controlling only mouse. This software provides alternative grid generation program to the existing expensive software from overseas but the potential of this software is not as much as the foreign software in term of more complicated boundary.

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Pitiya Namwong posses Bachelor Degree in Food Engineering from King Mongkut Institue of Technology Ladkrabang and Master Degree from Kasetsart University in Mechanical Engineering. At present, she is working at Effem Foods (Thailand) as a process engineer. She has been developing mathematical model by using C language on Visual C ++ program and also interested in mathematical model and Computational Fluid Dynamics research.

Asst. Prof. Dr. Varangrat Juntasaro posses Bachelor Degree and Ph.D. in Mechanical Engineering from Imperial College, University of London, United Kingdom. At present, she is working at department of Mechanical Engineering, faculty of Engineering, Kasetsart University, Bangkok, Thailand. Her research interests are Computational Fluid Dynamics (CFD), turbulence modeling, grid generation, parallel computing and clean room technology.