Robust Stabilization of Uncertain Linear System with Distributed State Delay*

Suthee phoojaruenchanachai, Kamol uahchinkul,
Jjongkol ngamwiwit and Yothin prempraneerach

ABSTRACT – In this paper, we present the theoretical development to stabilize a class of uncertain time-delay system. The system under consideration is described in state space model containing distributed delay, uncertain parameters and disturbance. The main idea is to transform the system state into an equivalent one, which is easier to analyze its behavior and stability. Then, a computational method of robust controller design is presented in two parts. The first part is based on solving a Riccati equation arising in the optimal control theory. In the second part, the finite dimensional Lyapunov min-max approach is employed to cope with the uncertainties. Finally, we show how the resulting control law ensures asymptotic stability of the overall system..

KEY WORDS -- Robust Stabilization, Time-delay System, Control System Design.

1. Introduction

It is well known that uncertain parameters and disturbance in practical control system such as modeling errors, linearization approximations, etc., are always found and disturb the system. On the other hand, the time delay which commonly encountered in various engineering systems is frequently a source of instability. Therefore, the problem of robust stabilization of state delayed system with uncertain parameters have received considerable attention of many researchers, and many solution approaches have been proposed, see, for example [1~9] and reference therein. One approach which frequently applied to deterministic robust control is by means of the so-called "second method" of Lyapunov. The design is based on a nominal linearization of given system with time-varying, non-linear uncertain elements of the system and the extraneous disturbances grouped into an unknown but bounded function. Only knowledge of compact sets bounding the system uncertainties is required. Furthermore, if they satisfy certain matching conditions, complete insensitivity to the system variations can be achieved, see, for example [1~8] and reference therein. Other often-used approach is based on H_∞ control theory of which resulting control law is linear [9][10]

In this paper, A class of linear system containing constant known distributed state delay, uncertain parameters and additive disturbances are presented. Determination of controller parameters can be divided into two parts. First, base on the improved Fiagbedzi and Pearson theorem [11~13], the linear transformation is utilized to reduce the original problem into an equivalent one which is easier to find the solution. Next, by using the well-known Lyapunov min-max approach of numerical example in section 4 demonstrates the proposed control method. Gutman [14], a suitable stabilizing control law is derived.

This paper is organized as follow. In the next section, we introduce the uncertain system considered here and the state-feedback based transformation technique. In section 3, we derive the required robust control law. A Some final remarks and conclusion appear in Section 5.

2. Problem Formulation

Consider a class of uncertain time-delay systems (S_d) which is defined by the following state equations

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$$\begin{aligned}\dot{x}(t) = & [A+\Delta A(t)]x(t)+[A_h+\Delta A_h(t)]x(t-h) \\ & +\int_{-r}^{0} A_r(\theta)x(t-r-\theta)d\theta \\ & +\int_{-r}^{0} \Delta A_r(t)x(t-r-\theta)d\theta \\ & +[B+\Delta B(t)]u(t)+Bw(t)\end{aligned} \tag{2.1}$$

where $x \in R^n$ is the current value of the system state, $u(t) \in R^m$ is the control function, $\omega(t) \in R^l$ is the additive disturbance A, A_h, B are known constant matrices of appropriate dimensions, $A_r \in L_1([-r,0];\ R^{n\times n})$ is a matrix whose elements are integrable functions on $[-r,0]$, $\Delta A(t)$, $\Delta A_h(t)$, $\Delta A_r(t)$, $\Delta B(t)$ are matrices whose elements are continuous, unknown but bounded functions, $h,\ r \in R^+$ is a known constant delay time and the initial function of system be specified as $x_0(\eta) \in C_d([-t_f,0], R^n)$, $t_f = \max(h,\ r)$.

ASSUMPTIONS

Before proposing our controllers, the following assumptions are made throughout here.

Assumption 2.1: The nominal system of (S_d); i.e., the system (S_d) which $\Delta A(t) = \Delta A_h(t) = \Delta A_r(t) = 0$, $\Delta B(t) = 0$, $w(t) = 0$ are spectrally stabilizable.

Assumption 2.2: For all $t \in R^+$, there are exist continuous matrix functions $H(t)$, $H_h(t)$, $H_r(t)$, and $E(t)$ of appropriate dimensions such that

a) $\Delta A(t) = BH(t)$,

b) $\Delta A_h(t) = BH_h(t)$,

c) $\Delta B(t) = BE(t)$,

d) $I + \frac{1}{2}\left(E(t)+E^T(t)\right) \geq \delta I$ for some scalar $\delta \rangle 0$,

e) $\Delta A_r(t) = BH_r(t)$

f) there are scalar $\mu(x_t)$ and $\mu_E(t)$ such that

$$\mu(x_t) \geq \left\| H(t)x(t)+H_h(t)x(t-h) + \int_{-r}^{0} H_r(t)x(t-r-\theta)d\theta + \omega(t) \right\|,$$

and

$$\mu_E(t) \geq \|E(t)\|$$

g) information δ, μ and μ_E are available

Note here that physical meaning of matching conditions defined in Assumption 2.2 is all disturbance and uncertainty effects disturb into the system though the same channel as the control. And if matching conditions are satisfies, we can rewrite system (S_d) to the form

$$\dot{x}(t) = Ax(t)+A_h x(t-h)+\int_{-r}^{0} A_r(\theta)x(t-r-\theta)d\theta) + B(u(t)+v(t)) \tag{2.2}$$

where

$$v(t) = H(t)x(t)+H_h(t)x(t-h) + \int_{-r}^{0} H_r(t)x(t-r-\theta)d\theta + E(t)u(t)+w(t) \tag{2.3}$$

Transformation Technique

We begin with the linear transformation T_c defined by

$$\begin{aligned} z(t) &= (T_c(x))(t) \\ &= x(t)+\int_{-h}^{0} e^{A_c\theta}A_h x(t-h-\theta)d\theta \\ &\quad +\int_{-r}^{0}\int_{-\theta}^{0} e^{A_c\tau}A_r(-r-\theta)x(t-r+\theta)d\tau d\theta \end{aligned} \tag{2.4}$$

where $A_c \in R^{n\times n}$ is a matrix yet to be defined.

***Proposition** 2.1* :

Let the matrix A_c be defined by

$$A_c = A + e^{-hA_c}A_h + \int_{-r}^{0} e^{-(r+\theta)A_c}A_r(\theta)d\theta, \tag{2.5}$$

and

$$\sigma_u(S_d) \subset \sigma(A_c) \subset \sigma(S_d) \tag{2.6}$$

where

$$\sigma(S_d) = \{s \in C;\ \det(sI - A - e^{-hs}A_h - \int_{-r}^{0} e^{-(r+\theta)s}A_r(\theta)d\theta) = 0\},$$

and

$$\sigma_u(S_d) = \{s \in \sigma(S_d);\ \ \mathrm{Re}(s) \geq 0\},$$

Then, $\dot{x}(t)$ satisfies (2.1) and hence (2.2), if and only if $\dot{z}(t)$ satisfies the system of the form (S_o)

$$\dot{z}(t) = A_c z(t) + B(u(t)+v(t)) \tag{2.7}$$

Furthermore, the following properties are true:

(a) $(A_c,\ B)$ is a stabilizable pair,

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(b) if $\lim_{t\to\infty}\|z(t)\| = 0$, then $\lim_{t\to\infty}\|x(t)\| = 0$

(c) if $\lim_{t\to\infty}\|z(t)\| \le k_1, \quad \exists k_1 < \infty$, then $\lim_{t\to\infty}\|x(t)\| \le k_2, \quad \exists k_2 < \infty$,

Proof:

By using the *Leibniz's formula* [15], it is straightforward to verify that (2.2) in conjunction with the transformation (2.4) yields (2.7); see Appendix. Property (a) follows from Theorem 3.2 of [8]. To show the property (b) and (c), are obtained using Laplace transform (2.4) to obtain, after some rearrangement,

$$X(s) = \Delta^{-1}(s)(sI - A_c)Z(s) + \Delta^{-1}(s)(sI - A_c)\Psi(s) \qquad (2.8)$$

where $\Delta(s) = \left\lfloor sI - A - e^{-hs}A_h \right\rfloor$, and

$$\Psi(s) = \int_{-h}^{0} e^{A_c\theta} A_h \int_{-(h+\theta)}^{0} e^{-s(\tau+h+\theta)} x_0(\tau)\,d\tau d\theta .$$

Next by setting $t = \tau + h + \theta$, Observe that

$$\int_{-(h+\theta)}^{0} e^{-s(\tau+h+\theta)} x_0(\tau)\,d\tau = \int_{0}^{(h+\theta)} e^{-st} x_0(t-h-\theta)\,dt$$

Since $x_0(\tau) = 0, \ \forall \tau \notin [-h, \ 0]$, we have

$$\int_{-(h+\theta)}^{0} e^{-s(\tau+h+\theta)} x_0(\tau)\,d\tau = \int_{0}^{\infty} e^{-st} x_0(t-h-\theta)\,dt = L\{x_0(t-h-\theta)\}.$$

where the operator $L\{.\}$ is defined as the Laplace Transform.

Furthermore, by setting $t = \tau + r + \alpha + \theta$,

$$\int_{-(r+\alpha+\theta)}^{0} e^{-s(\tau+r+\alpha+\theta)} x_0(\tau)\,d\tau = \int_{0}^{(r+\alpha+\theta)} e^{-st} x_0(t-r-\alpha-\theta)\,dt$$

Since $x_0(\tau) = 0, \ \forall \tau \notin [-t_f, \ 0]$, we have

$$\int_{-(r+\alpha+\theta)}^{0} e^{-s(\tau+r+\alpha+\theta)} x_0(\tau)\,d\tau = \int_{0}^{\infty} e^{-st} x_0(t-r-\alpha-\theta)\,dt = L\{x_0(t-r-\alpha-\theta)\}$$

This implies that

$$\begin{aligned}\psi(t) &= L^{-1}\{\Psi(s)\} \\ &= L^{-1}\left\{\int_{-h}^{0} e^{A_c\theta} A_h L\{x_0(t-h-\theta)\} d\theta\right\} \\ &\quad + L^{-1}\left\{\int_{-r}^{0}\int_{-(r+\theta)}^{0} e^{A_c\alpha} A_r(\theta) L\{x_0(t-r-\alpha-\theta)\} d\alpha d\theta\right\} \\ &= \int_{-h}^{0} e^{A_c\theta} A_h x_0(t-h-\theta)\,d\theta \\ &\quad + \int_{-r}^{0}\int_{-(r+\theta)}^{0} e^{A_c\alpha} A_r(\theta) x_0(t-r-\alpha-\theta)\,d\alpha d\theta\end{aligned}$$

and hence,

$$\psi(t) = 0, \ \forall t > t_f = \max(h, r).$$

Note here that (2.6) implies that all eigenvalues of the transfer function $\Delta^{-1}(s)(sI - A_c)$ are stable. Consequently, it can be verified that

$$\begin{aligned}\lim_{t\to\infty}\|x(t)\| &\le \lim_{t\to\infty}\left\|L^{-1}\{\Delta^{-1}(s)(sI - A_c)Z(s)\}(t)\right\| \\ &\quad + \lim_{t\to\infty}\left\|L^{-1}\{\Delta^{-1}(s)(sI - A_c)\Psi(s)\}(t)\right\| \\ &= \lim_{t\to\infty}\left\|L^{-1}\{\Delta^{-1}(s)(sI - A_c)Z(s)\}(t)\right\|.\end{aligned}$$

The above analysis imply that $\psi(t)$ does not influence stability of $x(t)$. Furthermore, asymptotic stability of $z(t)$ implies asymptotic stability of $x(t)$ since $\Delta^{-1}(s)(sI - A_c)$ is stable. □

3. Controller Design

In this section we present a suitable controller design, which is based on the Min-Max Approach of Gutman, to stabilize $z(t)$. Next we will proof that the controller can make the system asymptotically stable.

Theorem 3.1 : Suppose there exists a transforma-tion satisfying the hypothesis of proposition 2.1. Then, for given $Q > 0$, there exist a positive definite solution P to the Riccati equation

$$A_c^T P + PA_c - PBB^T P + Q = 0 \qquad (3.1)$$

Furthermore, a control law, which render closed-loop systems asymtotically stable, is given by

$$u(t) = u_L(t) + u_N(t) \qquad (3.2)$$

where

$$u_L(t) = -\frac{1}{2}B^T Pz(t) \qquad (3.3)$$

and

$$u_N(t) = -\frac{\rho^2(x_t)B^T Pz(t)}{\delta(\rho(x_t)\left\|B^T Pz(t)\right\| + e^{-\phi t})} \qquad (3.4)$$

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where the nonlinear gain

$$\rho(x_t) = \mu_E \|u_L(t)\| + \mu(x_t), \quad (3.5)$$

and $\phi \in R^+$ and δ is the positive scalar defined in Assumption 2-d.

Proof:
First, we take the positive definite function

$$V_z(t) = z^T(t)Pz(t) \quad (3.6)$$

as Lyapunov function candidate for the system (2.7) with control (3.2). Applying with the Riccati equation, the following is obtained of the derivative of V_z

$$\dot{V}_z = -z^T(t)\left[A_c^T P + PA_c\right]z(t) + 2z^T(t)PB(u(t)+v(t))$$

Substitute $u_L(t)$ in the above equality to get

$$\begin{aligned}\dot{V}_z &= -z^T(t)\left[A_c^T P + PA_c - PBB^T P\right]z(t) \\ &\quad + 2z^T(t)PB(u_N(t)+v(t)) \\ &= -z^T(t)Qz(t) + 2z^T(t)PB(u_N(t)+v(t))\end{aligned}$$

since $v(t)$ still has same value as defined in (2.3), we have

$$\dot{V}_z = -z^T(t)Qz(t) + \alpha(t) + \beta(t)$$

where

$$\alpha(t) = 2z^T(t)PB(I+E(t))u_N(t)$$

and

$$\beta(t) = 2z^T(t)PB\Big\{H(t)x(t) + H_h(t)x(t-h) + \int_{-r}^{0} H_r(t)x(t-r-\theta)d\theta + \omega(t) + E(t)u_L(t)\Big\}$$

Consequently from $u_N(t)$ defined in (3.4), $\alpha(t)$ becomes

$$\begin{aligned}\alpha(t) &= -2k(t)z^T(t)PB(I+E(t))B^T Pz(t) \\ &= -2k(t)z^T(t)PB\left(I + \frac{1}{2}(E^T(t)+E(t))\right)B^T Pz(t) \\ &\le -2k(t)z^T(t)PB\delta B^T Pz(t)\end{aligned}$$

where

$$k(t) = \frac{\rho^2(x_t)}{\delta(\rho(x_t)\|B^T Pz(t)\| + e^{-\phi t})} \quad (3.7)$$

Hence

$$\alpha(t) \le -2k(t)\delta\|B^T Pz(t)\|^2 \quad (3.8)$$

On the other hand,

$$\beta(t) \le 2\|B^T Pz(t)\|\rho(x_t) \quad (3.9)$$

Combining (3.8) with (3.9) to get

$$\alpha(t) + \beta(t) \le 2\Big(-k(t)\delta\|B^T Pz(t)\|^2 + \rho(x_t)\|B^T Pz(t)\|\Big) \quad (3.10)$$

(3.7) and (3.10) imply that

$$\begin{aligned}\alpha(t)+\beta(t) &\le 2\left(-\frac{\rho^2\|B^T Pz(t)\|^2}{\rho\|B^T Pz(t)\| + e^{-\phi t}} + \rho(x_t)\|B^T Pz(t)\|\right) \\ &\le 2\left(\frac{e^{-\phi t}\rho\|B^T Pz(t)\|}{\rho\|B^T Pz(t)\| + e^{-\phi t}}\right) \\ &\le 2e^{-\phi t}\end{aligned}$$

Therefore,

$$\begin{aligned}\dot{V}_z(t) &= -z^T(t)Qz(t) + \alpha(t) + \beta(t) \\ &\le -z^T(t)Qz(t) + 2e^{-\phi t}\end{aligned}$$

By using [17], It can be verified that closed-loop system (2.7) with control law (3.2) is asymptotically stable. □

4. Numerical Example and Simultion

Consider the following uncertain system with distributed time delay with its linear part as defined in

$$\dot{x}(t) = Ax(t) + [A_h + \Delta A_h(t)]x(t-h) + \int_{-r}^{0} A_r(\theta)x(t-r-\theta)d\theta + Bu(t) + \omega(t) \quad (4.1)$$

where

$$A = \frac{8e^{-\frac{\pi}{4}} - 8}{65},\ A_h = e,\ A_r(\theta) = \sin(8\theta),\ B = 1,$$

$$|\Delta A_h(t)| \le 3,\ |\omega(t)| =\le 0.5,\ h = 1,\ r = \frac{\pi}{4}$$

Therefore, it can be verified by using the numerical algorithm proposed in [15] that $\sigma_u = \{1\}$, consequently A_c has chosen to be 1. Next, we choose Q to be 3 then P becomes 3 and the nonlinear gain

$$\rho(x_t) = \mu(x_t) = \|3 \cdot x(t-h) + 0.5\| \quad (4.2)$$

Using the controller described and let ϕ be 0.15, the response of the system (4.1) with controller are given in the figure 1 shown below

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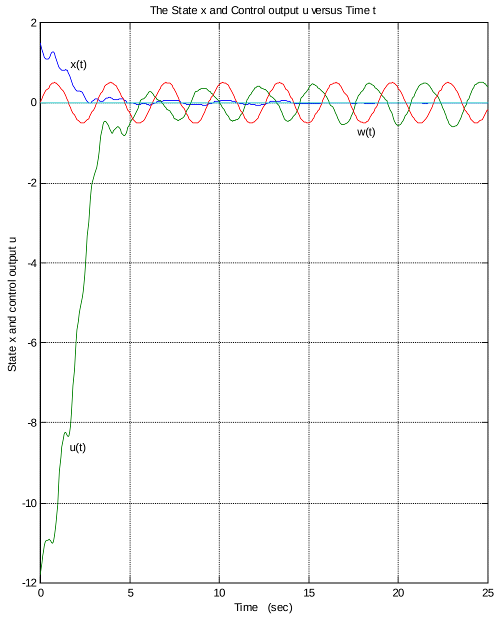

Fig 1. Simulation Result,

Furthermore, figure 2 shows the response of system (4.1) with only linear control law (3.3). It can be observed that the control law without nonlinear part can not stabilize the system. In the other hand, the nonlinear part is sufficient to control the effect of uncertainties and disturbance for stabilizing the system

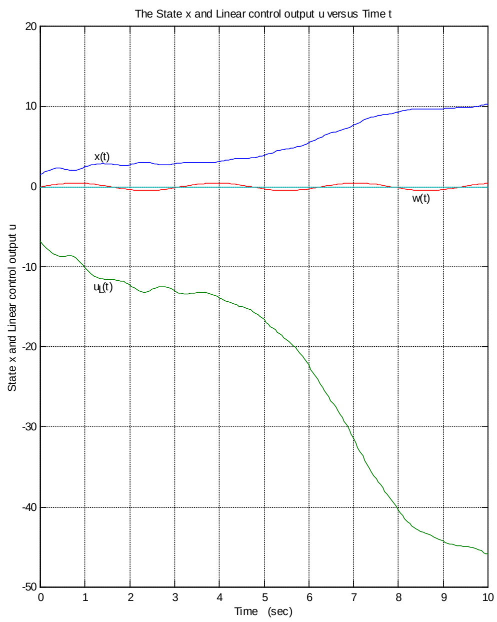

Fig 2. Simulation Result without nonlinear control law

5. Concluding Remarks

We have presented a method to stabilize uncertain system with distributed time delay. By using matching condition defined in *Assumption 2.2*, we can change system model (2.1) into new model (2.7) that easier to be applied with the special linear transformation. The transformation is used to reduce the system state model so that we can obtain a stabilizing condition as shown in *Proposition 2.1*. In the other word, choosing A_c, such that $\Delta^{-1}(s)(sI - A_c)$ stable, paves the way to feasible design of the robust controller. And it explains why stability of $z(t)$ can imply stability of $x(t)$. As a consequence, we obtain a new design of robustly stabilizing controller for the system.

In General, the problem of determining the roots of systems has been considered by many authors. Base on Manitus method [16], Fiagbedzi and Pearson had proposed the solution for distributed systems in 1987, see [13] for references.

Finally, we note that the applicability of our approach is not limited to stabilization problem. For instance, it is feasible to extend our result in section 3 to cope with the model-following control of which concept is proposed in [7].

6. APPENDIX

Supplementary proof of Proposition 2.1

From the hypothesis of Proposition 2.1, we have

$$\dot{x}(t) = Ax(t) + A_h x(t-h) + \int_{-r}^{0} A_r(\theta)x(t-r-\theta)d\theta) + B(u(t)+v(t)) \quad \text{(A.1)}$$

with the auxiliary output

$$z(t) = (T_c(x))(t) = x(t) + \int_{-h}^{0} e^{A_c\theta} A_h x(t-h-\theta)d\theta + \int_{-r}^{0}\int_{-\theta}^{0} e^{A_c\tau} A_r(-r-\theta)x(t-r+\theta)d\tau d\theta \quad \text{(A.2)}$$

where the matrix A_c be defined by

$$A_c = A + e^{-hA_c}A_h + \int_{-r}^{0} e^{-(r+\theta)A_c} A_r(\theta)d\theta, \quad \text{(A.3)}$$

By using the *Leibniz's formula* [12], it can be verified that

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$$\frac{d}{dt}\int_{-h}^{0} e^{A_c\theta} A_h x(t-h-\theta)d\theta$$
$$= e^{-hA_c} A_h x(t) - A_h x(t-h-\theta) + A_c \int_{-h}^{0} e^{A_c\theta} A_h x(t-h-\theta)d\theta$$

and

$$\frac{d}{dt}\int_{-r}^{0}\int_{-\theta}^{0} e^{A_c\tau} A_r(-r-\theta)x(t-r+\theta)d\tau d\theta$$
$$= \int_{-r}^{0}\Big[e^{-\theta A_c} A_r(-r-\theta)x(t) - A_r(-r-\theta)x(t+\theta) + A_c\int_{-\theta}^{0} e^{A_c\tau} A_r(-r-\theta)x(t-\tau+\theta)d\tau\Big]d\theta$$

Hence,

$$\begin{aligned}\dot z(t) &= \dot x(t) + \frac{d}{dt}\int_{-h}^{0} e^{A_c\theta} A_h x(t-h-\theta)d\theta + \frac{d}{dt}\int_{-r}^{0}\int_{-\theta}^{0} e^{A_c\tau} A_r(-r-\theta)x(t-r+\theta)d\tau d\theta\\
&= Ax(t) + A_h x(t-h) + \int_{-r}^{0} A_r(\theta)x(t-r-\theta)d\theta + B(u(t)+v(t))\\
&\quad + e^{-hA_c} A_h x(t) - A_h x(t-h-\theta) + A_c\int_{-h}^{0} e^{A_c\theta} A_h x(t-h-\theta)d\theta\\
&\quad + \int_{-r}^{0}\Big[e^{-\theta A_c} A_r(-r-\theta)x(t) - A_r(-r-\theta)x(t+\theta) + A_c\int_{-\theta}^{0} e^{A_c\tau} A_r(-r-\theta)x(t-\tau+\theta)d\tau\Big]d\theta\\
&= A_c\Big[x(t) + \int_{-h}^{0} e^{A_c\theta} A_h x(t-h-\theta)d\theta + \int_{-r}^{0}\int_{-\theta}^{0} e^{A_c\tau} A_r(-r-\theta)x(t-r+\theta)d\tau d\theta\Big]\\
&\quad + B(u(t)+v(t))\\
&\quad + \Big[A + e^{-hA_c} A_h + \int_{-r}^{0} e^{-(r+\theta)A_c} A_r(\theta)d\theta - A_c\Big]x(t)\end{aligned}$$

which is equivalent to

$$\dot z(t) = A_c z(t) + B(u(t)+v(t))$$

owning to (A.2) and (A.3).

Next, to show (2.8), Laplace transform (A.2) to obtain

$$\begin{aligned}Z(s) &= L\{z(t)\}\\
&= L\Big\{x(t) + \int_{-h}^{0} e^{A_c\theta} A_h x(t-h-\theta)d\theta + \int_{-r}^{0}\int_{-\theta}^{0} e^{A_c\alpha} A_r(-r-\theta)x(t-\alpha+\theta)d\alpha d\theta\Big\}\\
&= X(s) + \int_{-h}^{0} e^{A_c\theta} A_h L\{x(t-h-\theta)\}d\theta + \int_{-r}^{0}\int_{-\theta}^{0} e^{A_c\alpha} A_r(-r-\theta)L\{x(t-\alpha+\theta)\}d\alpha d\theta\end{aligned}$$

Consequently,

$$\begin{aligned}Z(s) &= X(s) + \int_{-h}^{0} e^{A_c\theta} A_h e^{-s(h+\theta)} d\theta X(s)\\
&\quad + \int_{-h}^{0} e^{A_c\theta} A_h \int_{-(h+\theta)}^{0} e^{-s(\tau+h+\theta)} x_0(\tau)d\tau d\theta\\
&\quad + \int_{-r}^{0}\int_{-(r+\theta)}^{0} e^{A_c\alpha} A_r(\theta)e^{-s(r+\alpha+\theta)} d\alpha d\theta X(s)\\
&\quad + \int_{-r}^{0}\int_{-(r+\theta)}^{0} e^{A_c\alpha} A_r(\theta)\int_{-(r+\alpha+\theta)}^{0} e^{-s(r+\alpha+\theta)} x_0(\tau)d\tau d\alpha d\theta\\
&= \Big[I + \int_{-h}^{0} e^{A_c\theta} A_h e^{-s(h+\theta)} d\theta + \int_{-r}^{0}\int_{-(r+\theta)}^{0} e^{A_c\alpha} A_r(\theta)e^{-s(r+\alpha+\theta)} d\alpha d\theta\Big]X(s)\\
&\quad + \int_{-h}^{0} e^{A_c\theta} A_h \int_{-(h+\theta)}^{0} e^{-s(\tau+h+\theta)} x_0(\tau)d\tau d\theta\\
&\quad + \int_{-r}^{0}\int_{-(r+\theta)}^{0} e^{A_c\alpha} A_r(\theta)\int_{-(r+\alpha+\theta)}^{0} e^{-s(r+\alpha+\theta)} x_0(\tau)d\tau d\alpha d\theta\end{aligned} \tag{A.4}$$

Note here that

$$I + \int_{-h}^{0} e^{A_c\theta} A_h e^{-s(h+\theta)} d\theta + \int_{-r}^{0}\int_{-(r+\theta)}^{0} e^{A_c\alpha} A_r(\theta)e^{-s(r+\alpha+\theta)} d\alpha d\theta = (sI - A_c)^{-1}\Delta(s) \tag{A.5}$$

where

$$\Delta(s) = \Big[sI - A - e^{-hs}A_h - \int_{-r}^{0} e^{-(r+\theta)A_c} A_r(\theta)d\theta\Big],$$

This can be verified easily by direct integration and then using (A.3). Finally, direct substitution (A.5) in (A.4) yields the required result.

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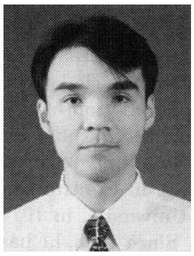

Suthee Phoojaruenchanachai graduated from King Mongkut's Institute of Technology Ladkrabang (KMITL), Bangkok in 1986. With scholarship suppport from the Japanese government, he received M.Eng. and D.Eng. from Tokyo Institute of Technology in 1990 and 1993 respectively, both in control engineering. Since August 1993 he has held a

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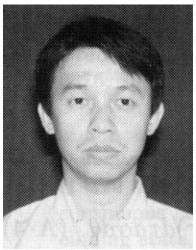
position of researcher at CTL. His research activities currently focus on information technology development for manufacturing automation and telemedicine. His responsibility is maimly project leading to develop prototypes of several products; e.g., calcium scoring software, medical image converter, and CNC simulator. As an invited lecture of KMITL, he also conducts course on stability theory and robust control. He has a broad research background with expertise in such areas as practical applications of modern control theory, development of electromechanical systems, computed tomagraphy and medical informatics. He has authored over 20 publications in these areas.



Kamol Uahchinkul was born in Bangkok in 1969. He received his B.Eng. in control Engineering and Master degrees in Electronical Engineering from King Mongkut's Institutute of Technology Ladkrabang (KMITL), Thailand in 1992 and 1998, respectively. Since July 1993, he has been with Product Research and Test Center, National Electronics and Computer Technology Center, Thailand, where he is currently an Associate Researcher. He is presently a Ph.D. candidate at the KMITL. His research interests include robust control, optimum control, adaptive control for delayed system.

Jongkol Ngamwiwit received her B.E. degree in electrical engineering from KMITL in 1972 and M.E. and Doctor of Engineering, from Tokai University, Japan in 1977 and 1987, respectively. Her research interacts include optimal control. She is a member of society of Instrument and Control Engineering (SICE). And now, she is Assoc. Prof. At Department of Control Engineering, Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang.

Yothin Prempraneerach received his Bachelor degreein Telecommunication Engineering from KMITL in 1971. He was awarded with the Japan International Coorperation Agency (JICA) scholarship and the Japan Society Promotion of Science (JSPS) scholarship to continue his furthur education in Japan and receive the M.S.E.E. and D.Eng. in Control Engineering from Nihoh University, in 1974 and 1983, respectively. Since 1983, he has been an Associate Professor at Department of Control Engineering, KMITL. Yothin has been conducting research and development in field of Control Engineering and received several research funds and awards from JICA and JSPS. Since 1992, he has been helping NECTEC in establishing of Electromagnetic Compatibility (EMC) Research and Service Laboratory in collaborated with KMITL. He has also been conducting several pioneer research and testing services in the field of EMC. During the establishment of laboratory, he has been visiting several EMC laboratories in many countries. Until 1997, the EMC Research and Service Laboratory underwent a transition from being one of NECTEC's laboratory to become the Product Research and Test Center (PTEC), he was then invited to be the director of PTEC.

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