Necessary Condition for Underground Hypocentre Location When Using the Least Square Error Approach

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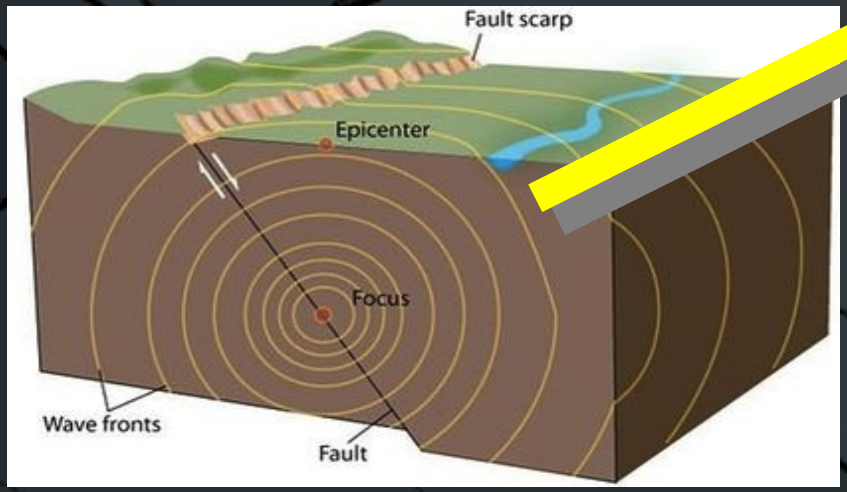
Seismic Station

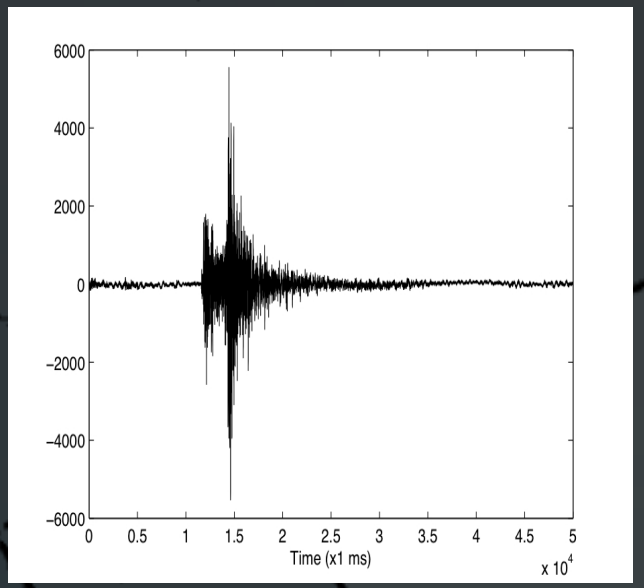
3. 65 Km → Find Location

2. Detected! → Alarm

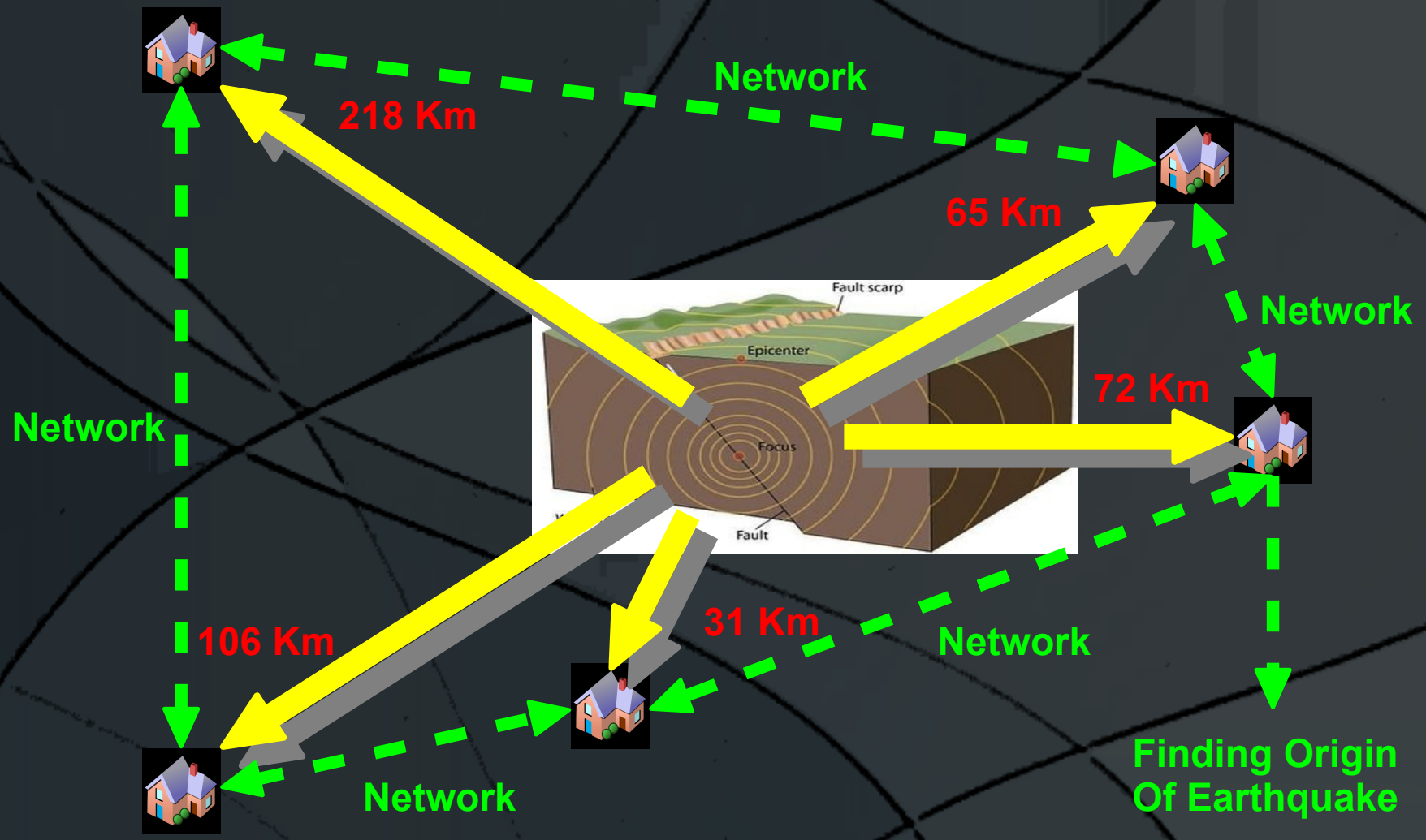
Seismic Station



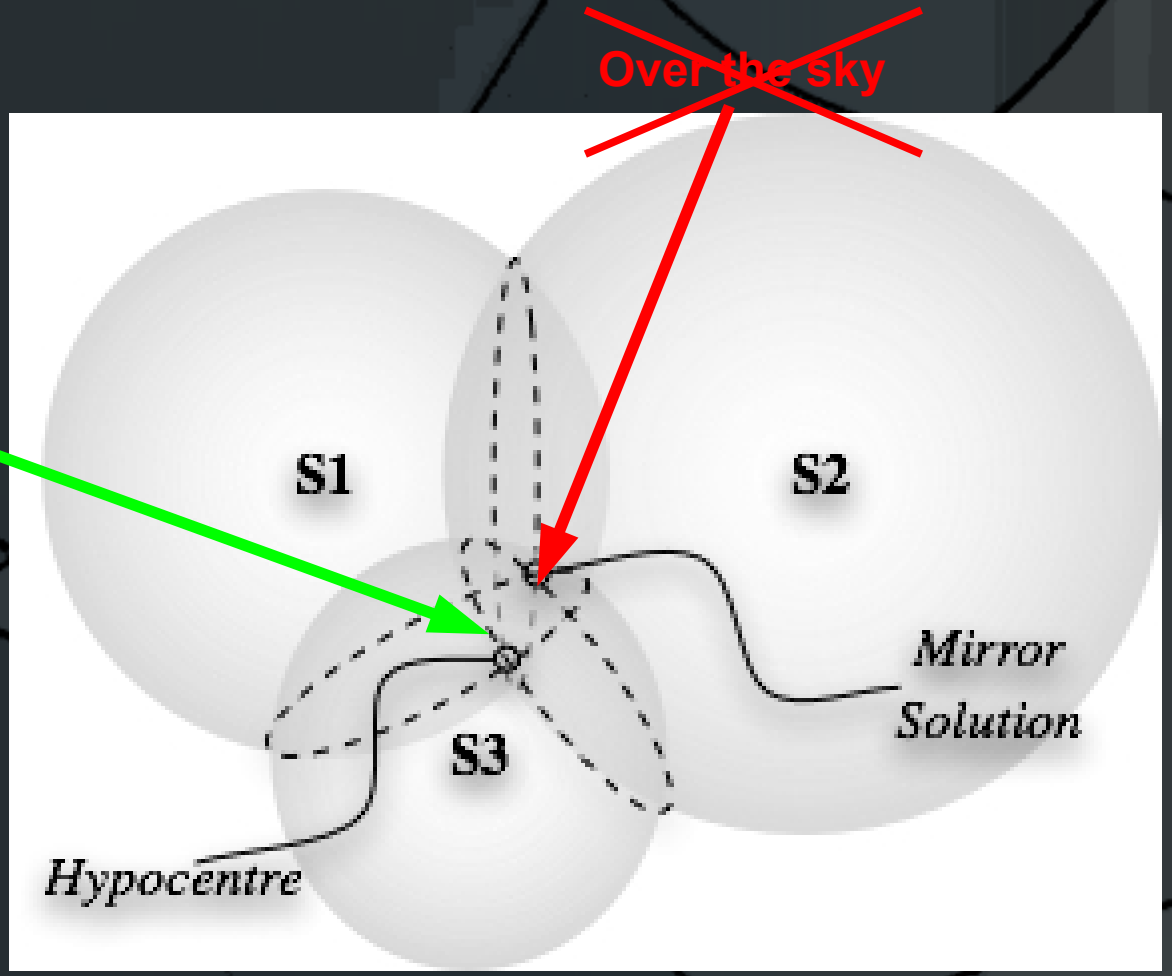
1. Earthquake



Earthquake Monitoring System



Hypocentre Location



$$r_i^2 = (x_i - \hat{x})^2 + (y_i - \hat{y})^2 + (z_i - \hat{z})^2$$

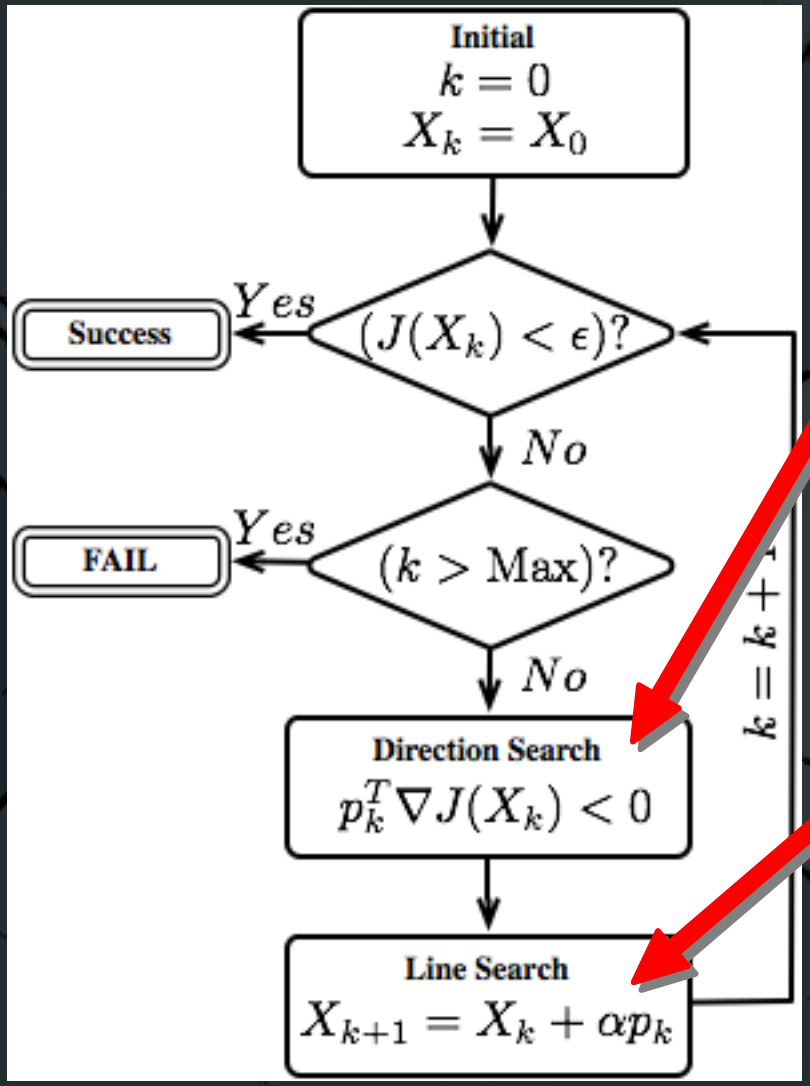
Least Square Error Method

$$J(x,y,z) = \sum_{i=0}^{n} \left((x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2 - r_i^2 \right)$$

Finding (x,y,z) that minimises $J(x,y,z)$

$J(x,y,z)=0 \rightarrow$ Hypocentre

Flowchart



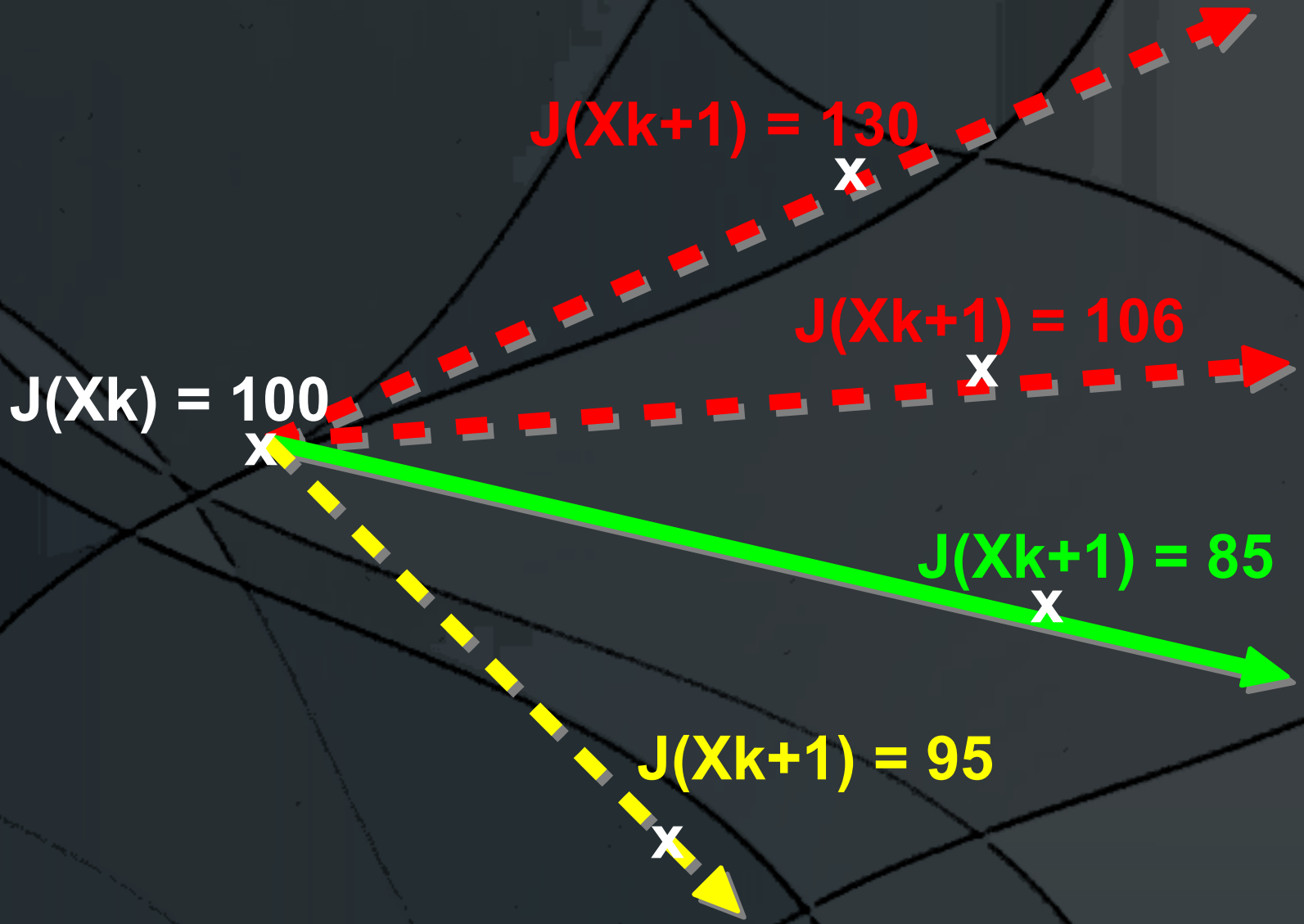
- Direction Search

  Find a guaranteed direction that makes J(*) decrease.

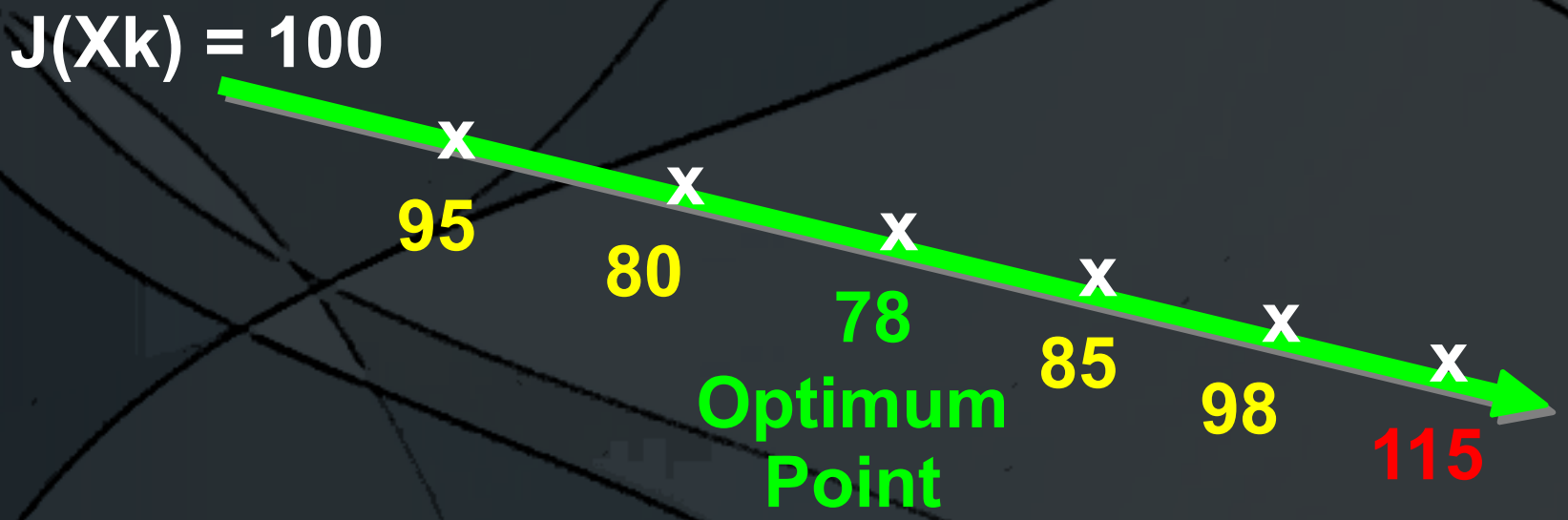
- Line Search

  Minimize J(*) in the above direction.

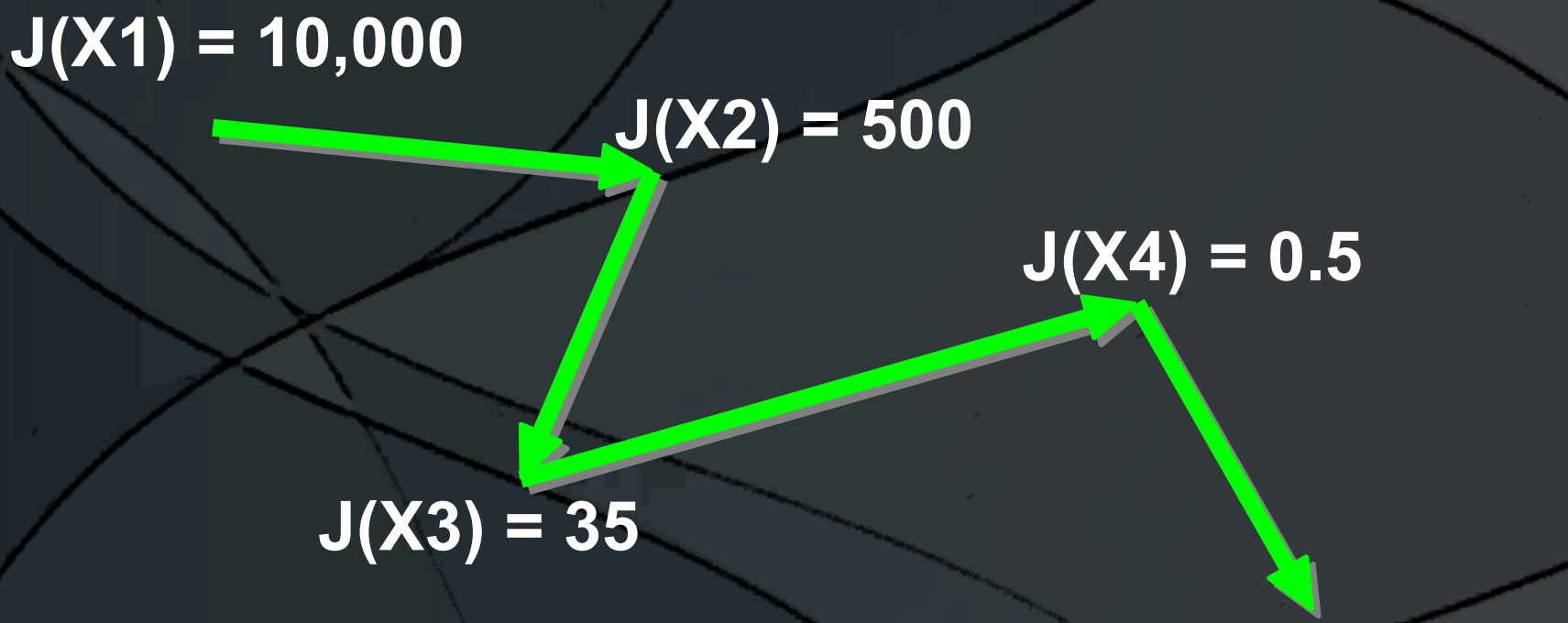
Direction Search



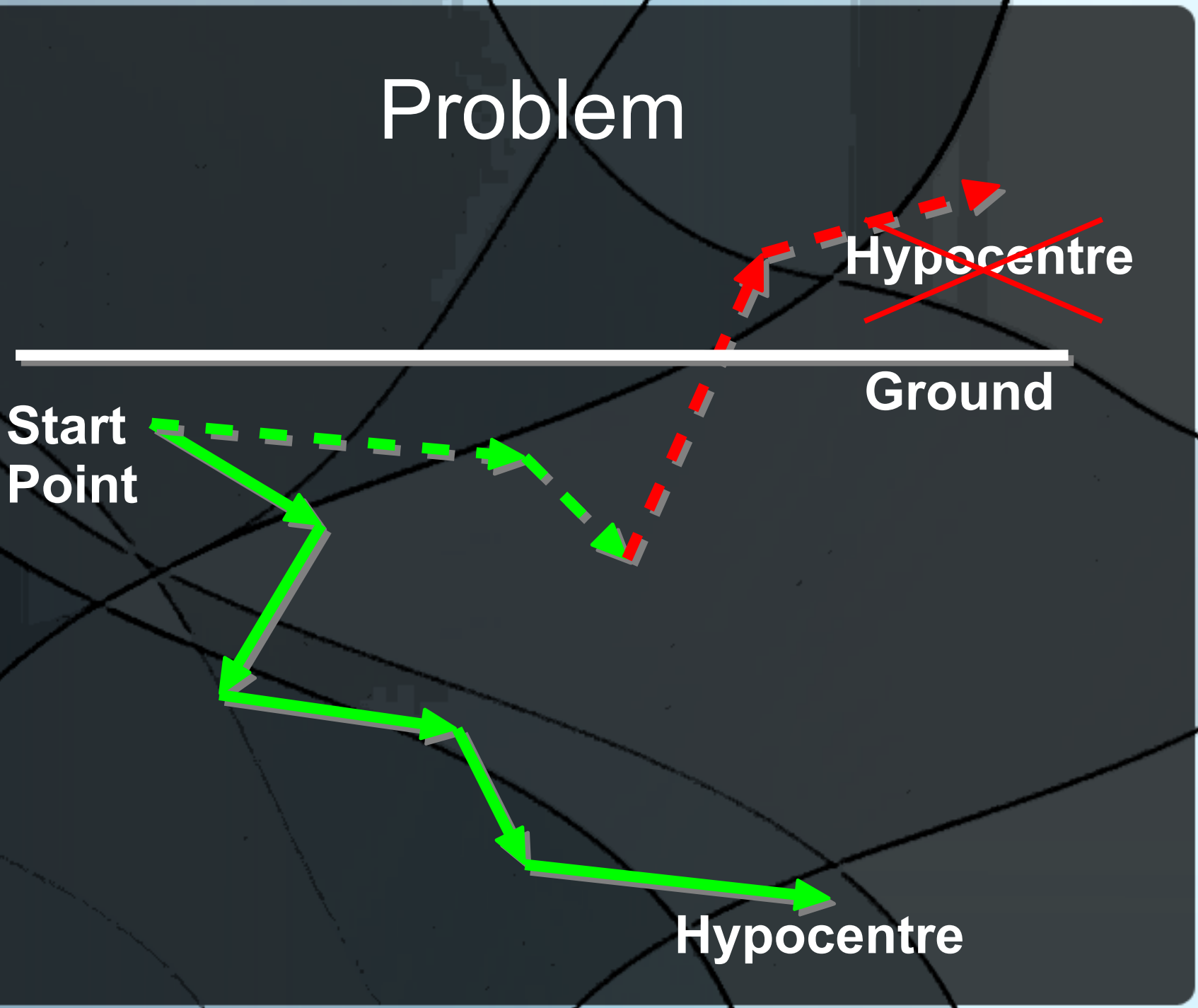
Line Search



Trajectory

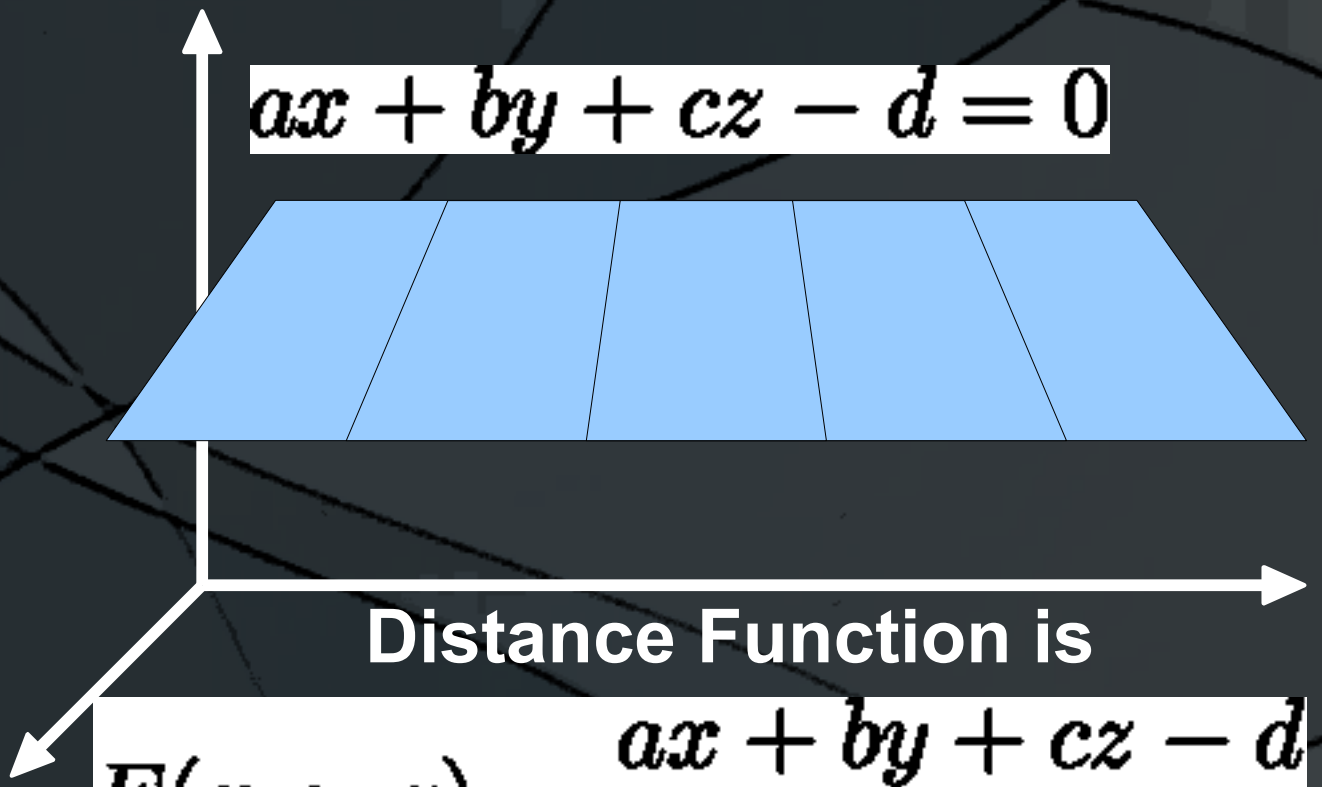


Problem



Proposed Necessary Condition

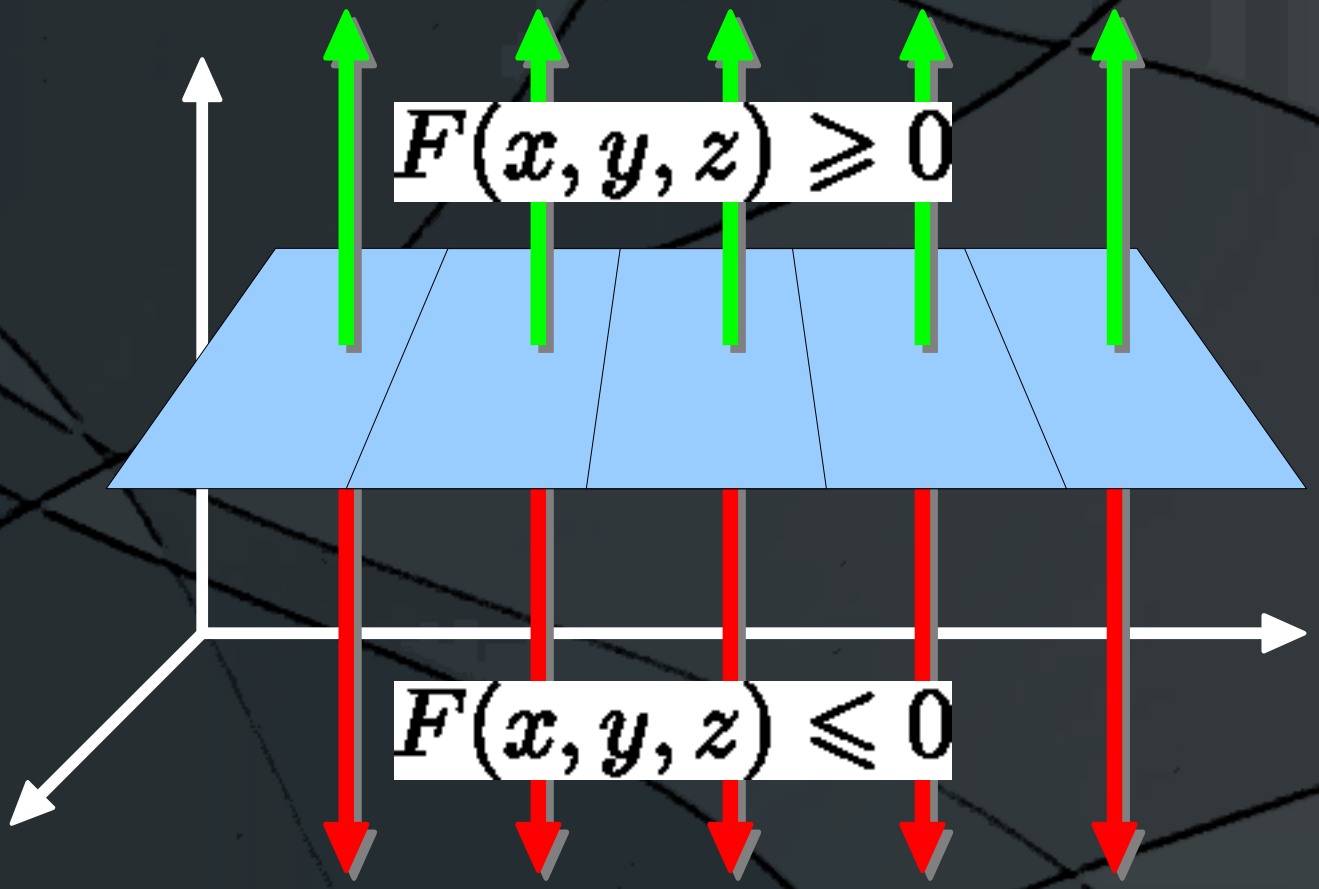
Auxiliary Plane



Distance Function is

$$F(x, y, z) = \frac{ax + by + cz - d}{\sqrt{a^2 + b^2 + c^2}}$$

Auxiliary Plane (2)

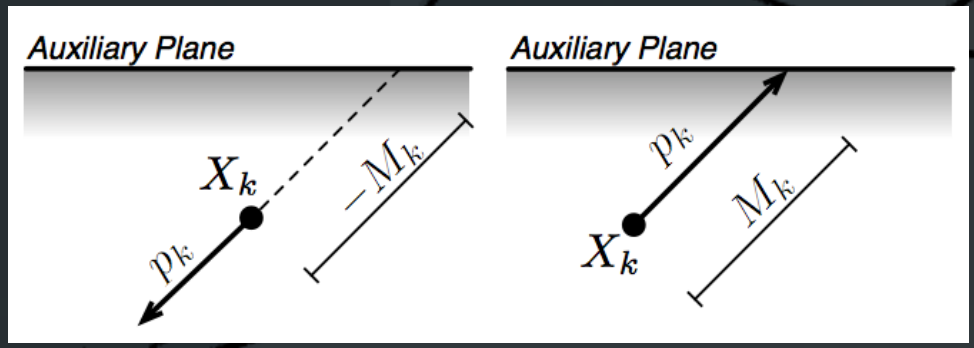


Necessary Condition

$$F_k(x, y, z) \cdot F_{k+1}(x, y, z) > 0$$

(x,y,z){k} **and** ***(x,y,z){k+1}*** **are in the same side.**

Apply on Line Search



Go downwards **Go towards plane**

$$
\begin{array}{rcl}
\textbf{if} & (A^T p_k)(F(X_k)) & \geqslant 0 \\
\textbf{then} & \alpha & > 0 > -M_k \\
\textbf{else} & M_k > \alpha & > 0 \\
\textbf{where} & M_k & = \left| \dfrac{F(X_k)||A||}{A^T p_k} \right|
\end{array}
$$

Simulation

Table 1. Seismic station information.

i	S_i	r_i(km)	error (km)
1	(0.00, 1125.00, 22.00)	633.03	+/- 1
2	(-556.00, -15.00, 5.00)	688.00	+/- 1
3	(524.00, 30.00, -2.00)	1132.75	+/- 1

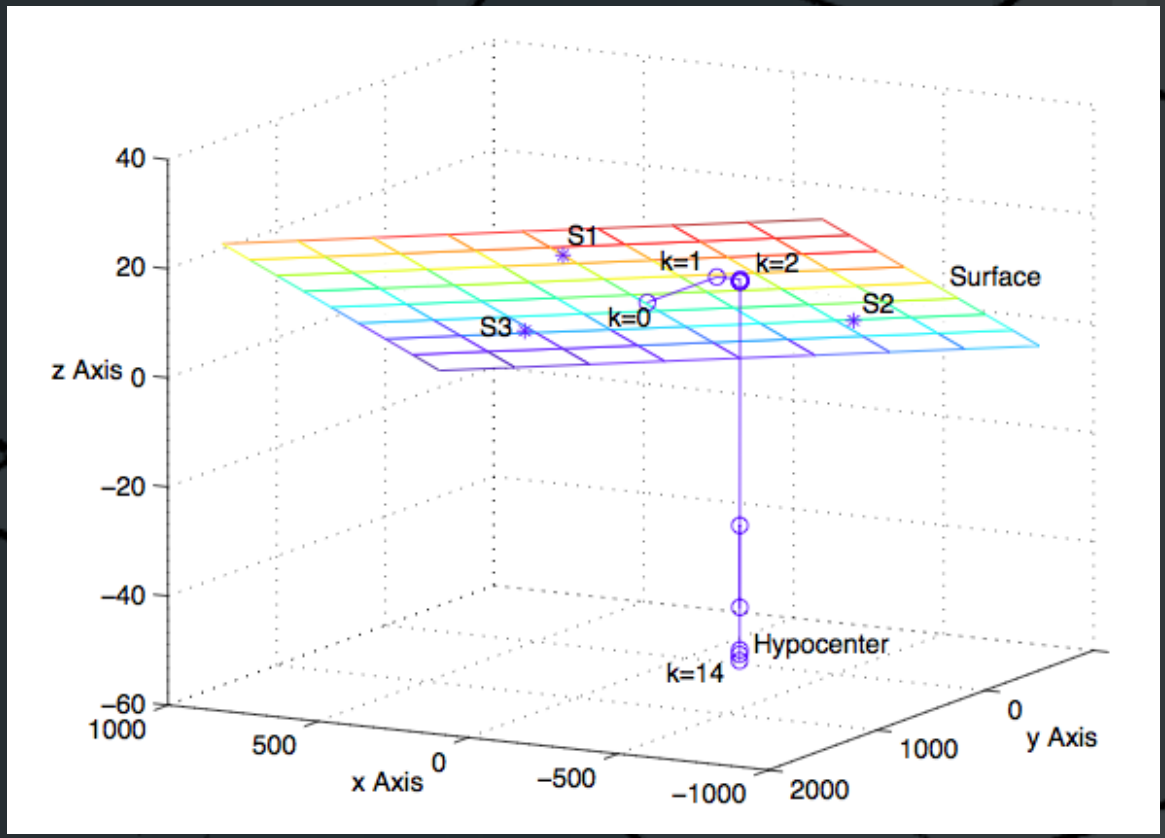
Actual Hypocentre: (-418, 656, -50)

Result

Table 2. Hypocentre estimation results.

k	X_k	$J(X_k)$	$F(X_k)$
0	(10.66, 380.00, -2.00)	7.896×10^{11}	-9.99
1	(-362.54, 711.99, 1.44)	1.926×10^{10}	-15.71
2	(-425.57, 666.37, 0.54)	6.947×10^{8}	-16.26
3	(-425.59, 666.31, 4.61)	6.946×10^{8}	-12.19
4	(-419.77, 656.97, -0.91)	3.522×10^{7}	-17.50
5	(-420.92, 696.14,-11.44)	2.968×10^{7}	-28.01
6	(-420.71, 656.24,-34.57)	1.558×10^{7}	-51.15
7	(-419.95, 656.41,-56.57)	1.209×10^{7}	-73.14
8	(-418.90, 656.75,-49.39)	1.758×10^{6}	-65.96
9	(-418.31, 656.38,-55.91)	9194.032	-72.46
10	(-418.35, 656.33,-55.86)	43.87908	-72.42
11	(-418.35, 656.34,-55.89)	0.21589	-72.44
12	(-418.35, 656.34,-55.88)	0.18189	-72.43
13	(-418.35, 656.34,-55.88)	0.00000	-72.44
14	(-418.35, 656.34,-55.88)	0.00000	-72.44

Trajectory


S1
k=1
k=2
Surface
S2
S3
k=0
Hypocenter
k=14
z Axis
40
20
0
−20
−40
−60
1000
500
0
−500
−1000
x Axis
0
1000
2000
y Axis

Thank You

Q & A