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Optimizing Weight Factors in Multi-Objective Geometric Programming

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Outline

- Introduction.
- Single-objective vs multi-objective optimization.
- Geometric programming (GP).
- Op-amp design via GP.
- Single-objective vs multi-objective GP.
- Proposed MOGP algorithm.
- MOGP: fixed weights vs optimized weights.
- Conclusions.

Mathematical Programming Optimization

Minimize f(x) such that

$g_i(x) \leq c$; i = 1…m

$h_i(x) = c$; i = 1…n

x is vector of variables.

- Linear program: f(x),$g_i(x)$,$h_i(x)$ are linear functions, eg. Ax+B.
- Quadratic program: f(x) is quadratic function and $g_i(x)$, $h_i(x)$ are linear functions.
- Integer program: same as linear program but x must be integer-valued.

Single-Objective vs Multi-Objective Optimization

- Single-objective $\Rightarrow$ f(x) represents one objective.
- Multi-objective $\Rightarrow$ f(x) represents a set of objectives.

$$\begin{aligned} \text{Minimize} \quad & F(x) = \left[f_1(x), \ldots\ldots , f_n(x) \right] \\ \text{subject to} \quad & G_i(x) = 0, \quad i = 1, \ldots, m \\ & G_j(x) \le 0, \quad j = 1, \ldots, p \\ & x_l \le x \le x_u \end{aligned}$$

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Solving Multi-Objective Problem

- Scalar method : combine multiple objectives into one scalar objective, eg. weighted sum.

$$\Rightarrow \text{ minimize } \quad \Sigma\, w_i\, f_i(x)$$

Geometric Programming

Minimize f(x) such that

$g_i(x) \leq 1 \quad ; \; i = 1 \ldots m$

$h_i(x) = 1 \quad ; \; i = 1 \ldots n$

$x_i > 0 \quad ; \; i = 1 \ldots p$

- f(x) and $g_i(x)$ are posynomial functions; $h_i(x)$ are monomial functions.
- Posynomial functions follow the following form

$$f(x_1, \ldots, x_n) = \sum_{k=1}^{t} c_k x_1^{\alpha_{1k}} x_2^{\alpha_{2k}} \ldots x_n^{\alpha_{nk}} \qquad ; c_k \geq 0$$

- Monomial functions are posynomials with only one term.

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Multi-Objective Geometric Program

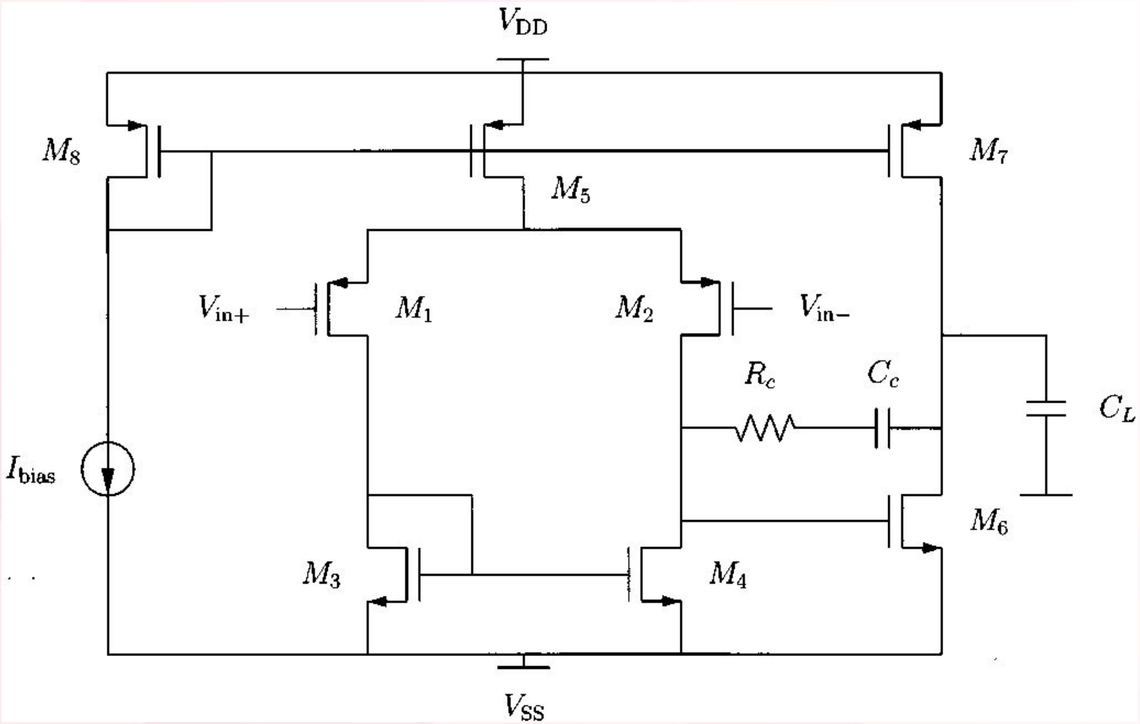
- Based on scalar formulation such as weighted-sum or product, new combined objective is also posynomial as posynomials are closed under positive additions and multiplications.

$$\text{Minimize} \quad F_s(x) = \sum w_i f_i(x) \;\; ; w_i \geq 0$$

$$\text{Minimize} \quad F_p(x) = \prod f_i(x)$$

- Both $F_s(x)$ and $F_p(x)$ are also posynomials.

Example: 2-Stage Op-Amp Design



- Differential-pair input stage, frequency-compensation network, output-stage driver.

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Single-Objective GP

I. Maximize UGBW (= minimize 1/UGBW)
II. Maximize DCgain (= minimize 1/DCgain)
III. Minimize Noise
IV. Minimize Power

subject to

- Symmetry and matching: *M1=M2, M3=M4*
- Limit on device sizes: $W \geq Wmin$, $L \geq Lmin$
- Limit on chip area: $A \leq Amax$
- Systematic input offset voltage
- Current ratio equalities: $I(M5) \alpha I(M8)$, $I(M7) \alpha I(M8)$, $I(M1) \alpha I(M5)$
- Bias conditions: $Vgs\text{-}Vt \leq Vds$
- Gate overdrive voltage: $Vgs\text{-}Vt \geq Vod,min$
- Limit on power consumption: $P \leq Pmax$
- Open-loop DC gain
- Unity-gain bandwidth
- Phase-margin
- Slew-rate
- Common-mode rejection ratio
- Power-supply rejection ratio
- Input-referred noise

GP Implementation

- Total of 46 constraints expressed by posynomials and monomials.

- Total of 19 design variables (W,L of transistors, R,C of frequency compensation network, and bias current).

- Optimization run time < 2 sec for each objective.

Optimization Results (SOGP)

Performance Measure	Specification	Design Objective			
		Max. UGBW	Max. DC gain	Min. noise	Min. power
Device length (μm)	≥ 0.8	0.8 (min)	0.8 (min)	0.8 (min)	0.8 (min)
Device width (μm)	≥ 2.0	2.0 (min)	2.0 (min)	2.0 (min)	2.0 (min)
Area (μm^2)	≤ 10000	7283	9162	10000	7218
Capacitance size (pF)	0.1 ≤ C ≤ 2000	2.68	2.7	3.86	2.68
Load capacitance (pF)	3	3	3	3	3
Common-mode input range (V)	includes 0.5Vdd	includes 0.5Vdd	includes 0.5Vdd	includes 0.5Vdd	includes 0.5Vdd
Output voltage range (V)	[0.1,0.9]Vdd	[0.028,0.91]Vdd	[0.018,0.9]Vdd	[0.026,0.904]Vdd	[0.024,0.908]Vdd
Power (mW)	≤ 5	5	5	5	***3.9***
DC gain (dB)	≥ 80	89.4	***95.8***	91.7	91.5
Unity-gain BW (MHz)	≥ 80	***90.1***	80	80	80
Phase margin (°)	≥ 60	60	60	60	60
Slew rate (V/μs)	≥ 10	87.2	53.5	61.4	68.5
CMRR (dB)	≥ 60	92.6	99.1	95	94.7
Neg. PSRR (dB)	≥ 80	98.5	105	100.9	100.6
Pos. PSRR (dB)	≥ 80	118.5	124.9	120.8	120.6
Input-referred noise, @1KHz (nV/√Hz)	≤ 300	300	244.7	***209***	300

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Multi-Objective GP

- In contrast to single-objective formulation, several desired objectives can be optimized simultaneously.
- Weight factors can be assigned to each objective to quantify its significance.
- Normalization is needed to account for the difference in units of individual objectives.
- Normalization factors can be readily determined by performing a single-objective optimization excluding the other objectives.

Multi-Objective formulation

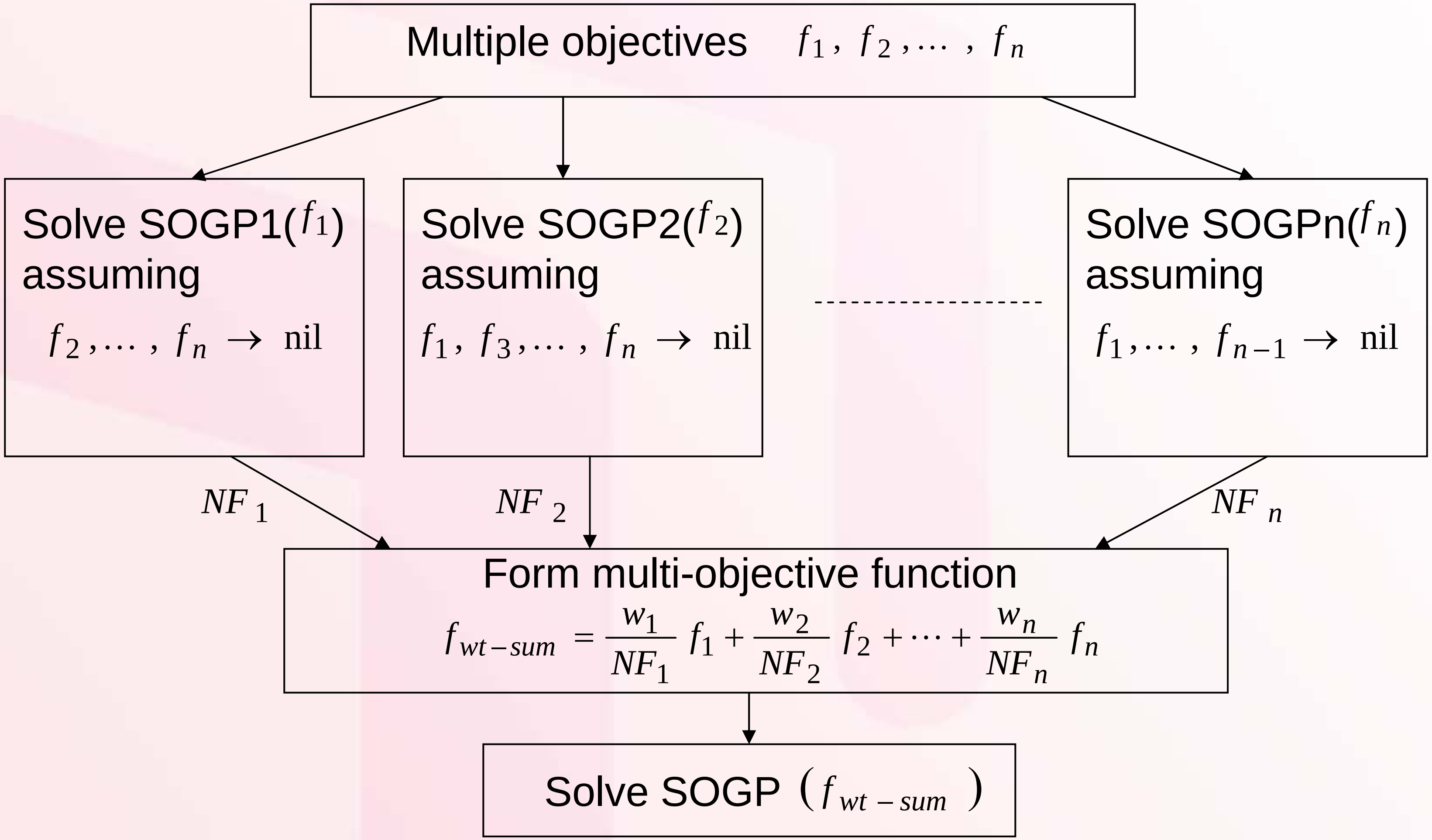
- Weighted-sum formulation

$$\text{Minimize } \left\{ w_1 \cdot \left(\frac{1}{BW_{norm}} \right) + w_2 \cdot \left(\frac{1}{Gain_{norm}} \right) + w_3 \cdot Noise_{norm} + w_4 \cdot Power_{norm} \right\}$$

- Product formulation

$$\text{Minimize } \left\{ \left(\frac{1}{BW} \right) \cdot \left(\frac{1}{Gain} \right) \cdot Noise \cdot Power \right\}$$

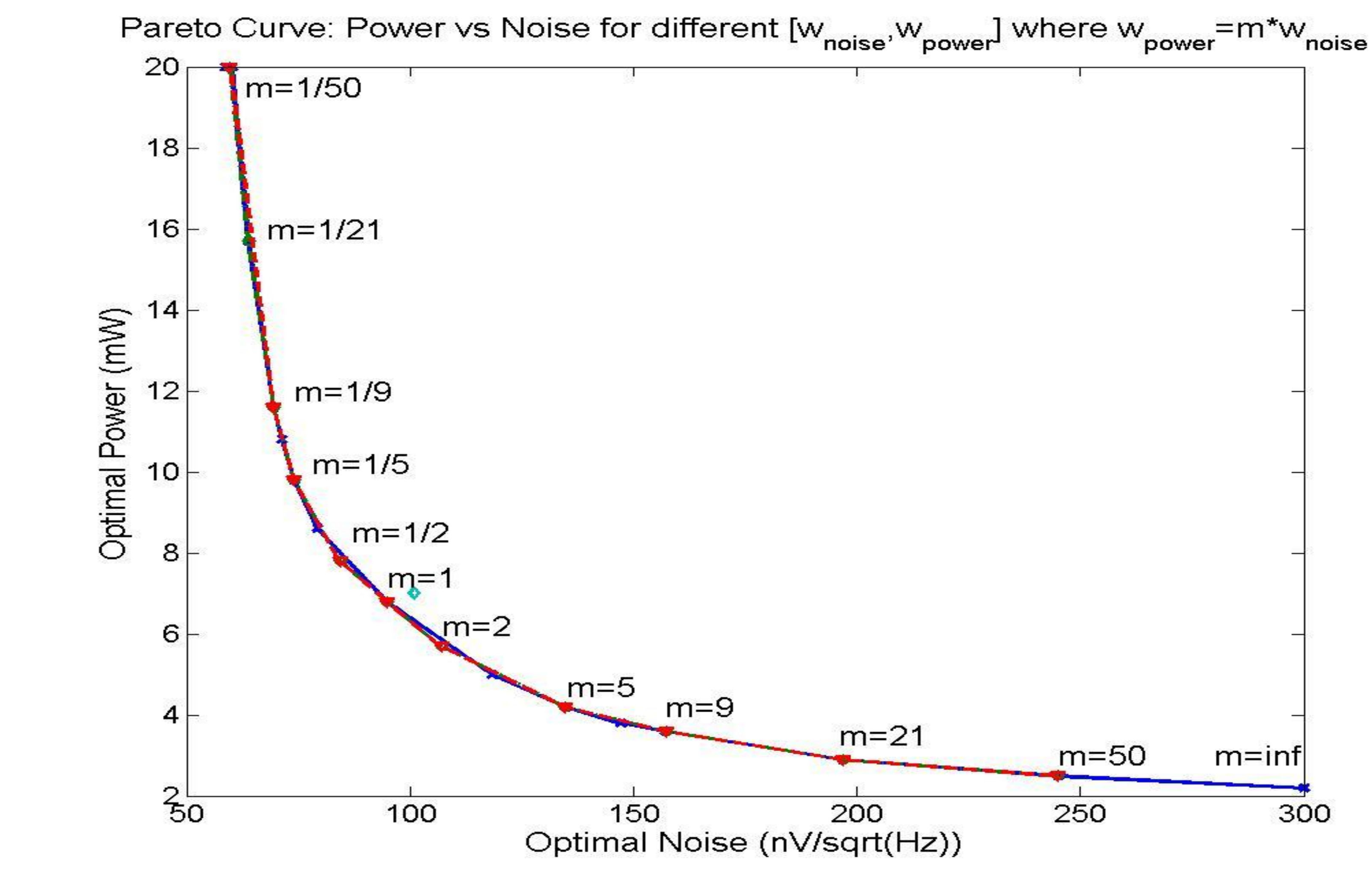
Proposed Algorithm for MOGP (weighted sum)



Optimization Results (MOGP)

Performance Measure	Specification	Weighted sum w=[1/4, 1/4, 1/4, 1/4]	Weighted sum w=[1/6, *1/2*, 1/6, 1/6]	Weighted sum w=[1/6, 1/6, *1/2*, 1/6]	Product
Device length (μm)	≥ 0.8	0.8 (min)	0.8 (min)	0.8 (min)	0.8 (min)
Device width (μm)	≥ 2.0	2.0 (min)	2.0 (min)	2.0 (min)	2.0 (min)
Area (μm²)	≤ 10000	10000	10000	10000	10000
Capacitance size (pF)	0.1 ≤ C ≤ 2000	3.3	3.2	3.6	3.3
Load capacitance (pF)	3	3	3	3	3
Common-mode input range (V)	includes 0.5Vdd	includes 0.5Vdd	includes 0.5Vdd	includes 0.5Vdd	includes 0.5Vdd
Output voltage range (V)	[0.1,0.9]Vdd	[0.02,0.9]Vdd	[0.02,0.9]Vdd	[0.02,0.9]Vdd	[0.02,0.9]Vdd
Power (mW)	≤ 5	5	5	5	5
DC gain (dB)	≥ 80	95.5	***95.7***	94	95.5
Unity-gain BW (MHz)	≥ 80	80	80	80	80
Phase margin (°)	≥ 60	60	60	60	60
Slew rate (V/μs)	≥ 10	51	52.3	54.4	51
CMRR (dB)	≥ 60	98.7	99	97.2	98.8
Neg. PSRR (dB)	≥ 80	104.6	104.8	103.1	104.7
Pos. PSRR (dB)	≥ 80	124.6	124.8	123.1	124.6
Input-referred noise, @1KHz (nV/√Hz)	≤ 300	224.6	228.9	***214.3***	225

Pareto Front (Trade-Off Curve)



MOGP with Weight Optimization

- Arbitrary weight assignment can lead to a solution far from "ideal multi-objective optimum," defined as the optimum achieved when each individual objectives reaches its own optimum simultaneously, i.e.

$$f_{wt-sum} = \frac{w_1}{NF_1} f_1 + \frac{w_2}{NF_2} f_2 + \cdots + \frac{w_n}{NF_n} f_n$$

When $f_i = NF_i$

$$f_{wt-sum,ideal} = w_1 + w_2 + \cdots + w_n$$

Solving MOGP with Weight Optimization

$$f_{wt-sum} = \frac{w_1}{NF_1} f_1 + \frac{w_2}{NF_2} f_2 + \cdots + \frac{w_n}{NF_n} f_n$$

- Use the same algorithm as fixed-weight MOGP but, now, w's are treated as additional variables.

- Need to introduce additional constraint on weights.

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Geometric-Mean Constraint

- To take into account of weight factors into MOGP, we introduce an additional constraint on weight factors, ie. the geometric mean of the weight factors equal unity as follows:

$$\left(w_1 \cdot w_2 \cdot \cdots \cdot w_n\right)^{1/n} = 1$$

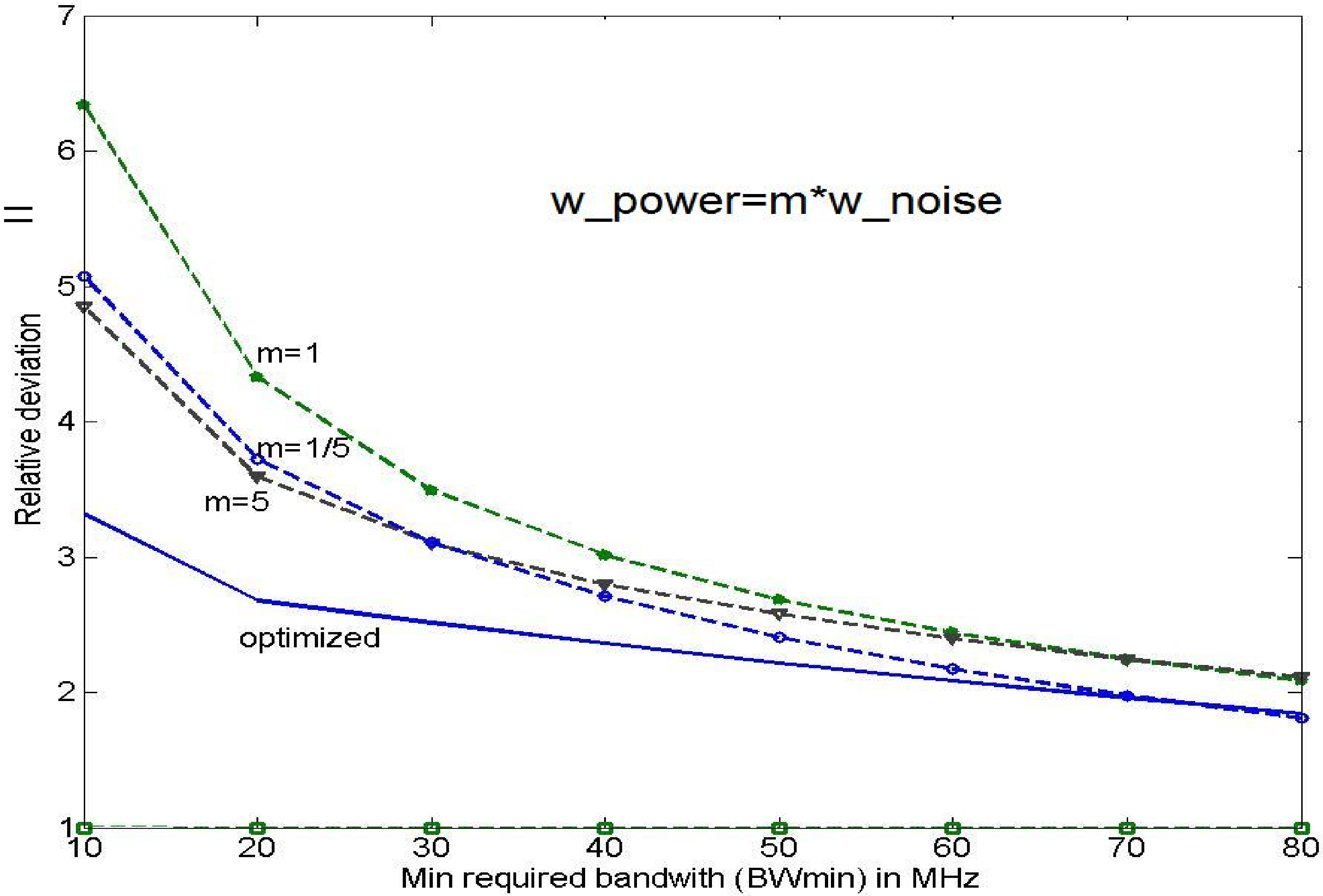
- The constraint is necessary to obtain a valid solution, similar to the unity-arithmetic-mean constraint imposed in the fixed, arbitrarily-assigned weights. Geometric mean is chosen, instead, b/c of its monomial form.

Optimization Results (MOGP w/ Weight Optimization)

Performance Measure	Specification	Weighted sum w=[1/4, 1/4, 1/4, 1/4]	Optimized-weight sum w_{opt}=[0.9, 2.1, 0.8, 0.7]
Device length (μm)	≥ 0.8	0.8 (min)	0.8 (min)
Device width (μm)	≥ 2.0	2.0 (min)	2.0 (min)
Area (μm^2)	≤ 40000	40000	40000
Capacitance size (pF)	0.1 ≤ C ≤ 2000	15	14
Load capacitance (pF)	3	3	3
Common-mode input range (V)	includes 0.5Vdd	includes 0.5Vdd	includes 0.5Vdd
Output voltage range (V)	[0.1,0.9]Vdd	[0.01,0.9]Vdd	[0.01,0.9]Vdd
Power (mW)	≤ 20	***6.8***	***7***
DC gain (dB)	≥ 80	***102***	***104***
Unity-gain BW (MHz)	≥ 60	***60***	***60***
Phase margin (°)	≥ 60	60	60
Slew rate (V/μs)	≥ 1	25.4	23
CMRR (dB)	≥ 60	105	107
Neg. PSRR (dB)	≥ 80	111	113
Pos. PSRR (dB)	≥ 80	131	133
Input-referred noise, @1KHz (nV/√Hz)	≤ 300	***95***	***101***

Relative deviation vs BWmin

$$\frac{f_{wt\_sum}}{f_{wt\_sum,ideal}} =$$



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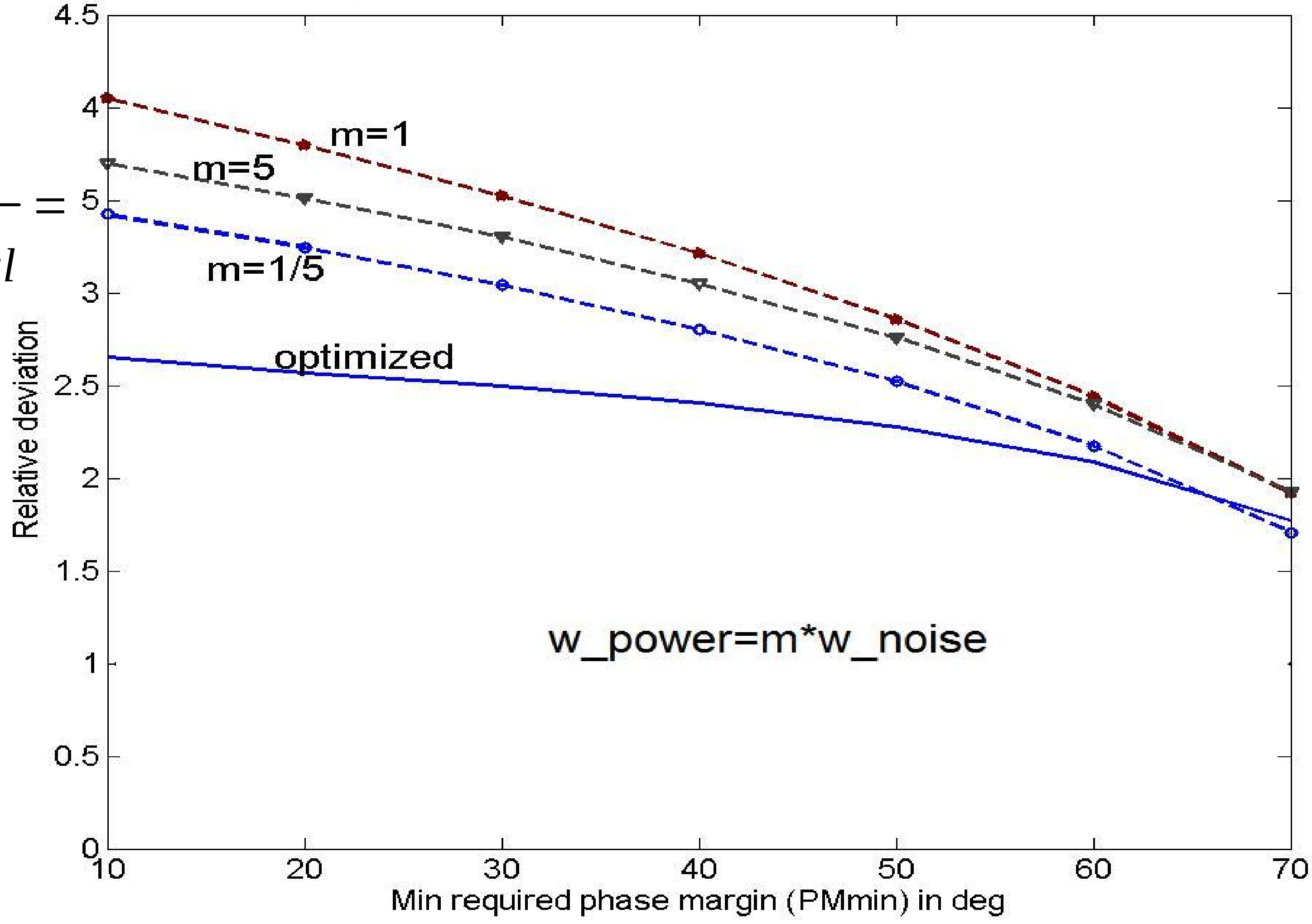
Relative deviation vs PMmin

$$\frac{f_{wt\_sum}}{f_{wt\_sum,ideal}} =$$



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Fixed W's (m=1) vs Optimized W's

$$\frac{f_{wt\_sum}}{f_{wt\_sum,ideal}} =$$

Bandwidth	Fixed equal weights	Optimized weights	Difference
60 MHz	2.4 x	2.1 x	14%
40 MHz	3.0 x	2.4 x	25%
20 MHz	4.3 x	2.7 x	59%

Phase margin	Fixed equal weights	Optimized weights	Difference
60 deg	2.4 x	2.1 x	14%
40 deg	3.2 x	2.4 x	33%
20 deg	3.8 x	2.6 x	46%

Conclusions

- Geometric program can be used to performed multi-objective design optimization.

- Proposed algorithm for MOGP has been presented.

- Contrary to conventional MOGP, weight factors can be taken into the optimization, yielding a solution closer to the ideal multi-objective optimum than the fixed, arbitrarily-assigned weights.