Data Fusion Based on Dempster-Shafer Theory with Interval Analysis for Decision Making

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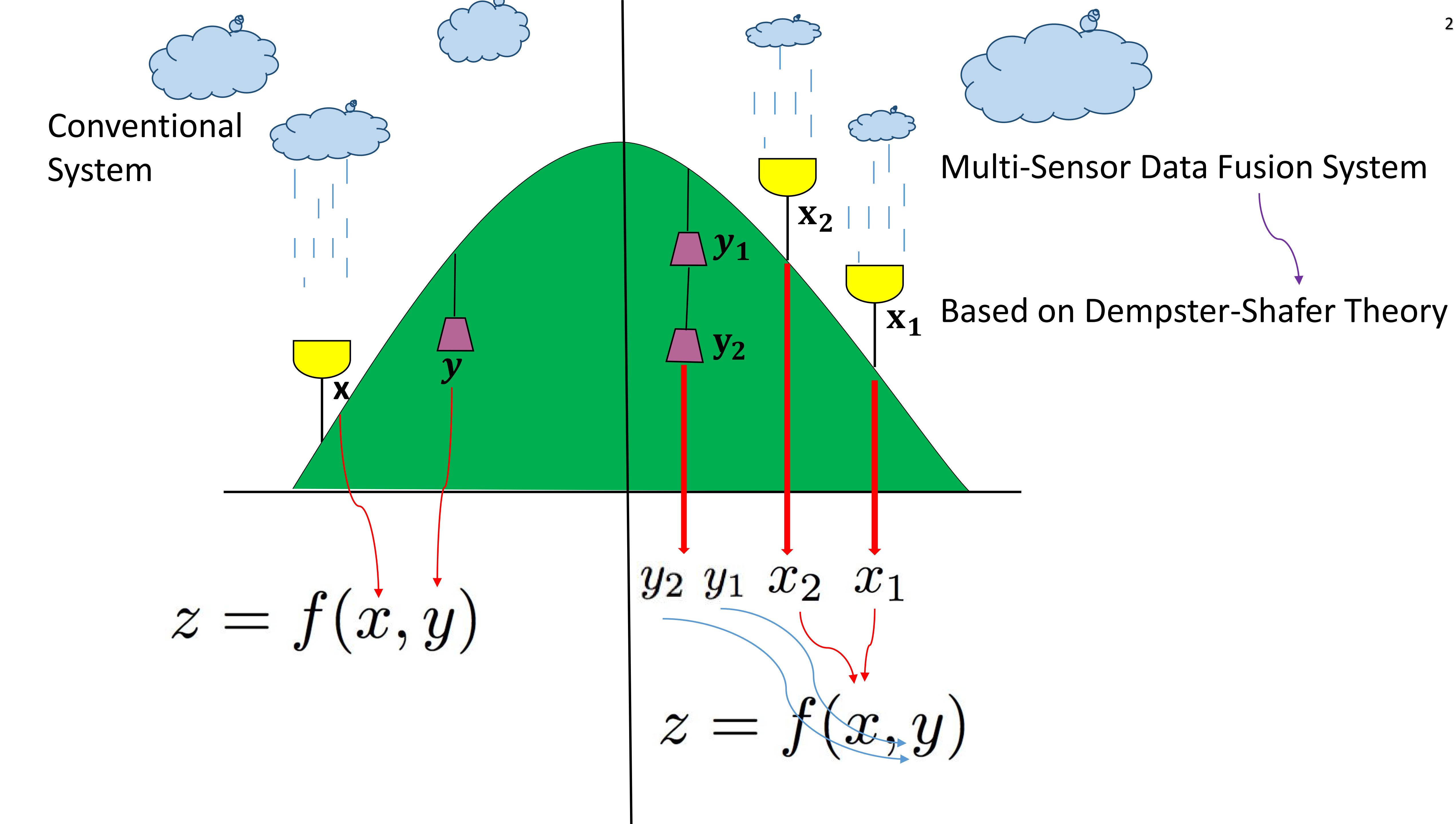
AIM

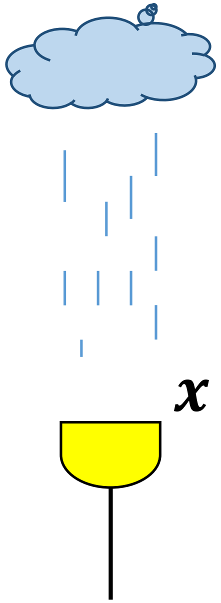
To introduce a technique that we can use to combine our knowledge from different sources in order to improve the accuracy of predicting or estimating results from prediction or estimation system


Conventional System
Multi-Sensor Data Fusion System
Based on Dempster-Shafer Theory
x
y
y_1
y_2
x_2
x_1
$z = f(x, y)$
y_2 y_1 x_2 x_1
$z = f(x, y)$

Interval Analysis

- We represent our knowledge about x variable by an interval [x]

Any binary operators can be extended to intervals.



$$[x] + [y] = [\underline{x} + \underline{y}, \overline{x} + \overline{y}]$$

$$[x] - [y] = [\underline{x} - \overline{y}, \overline{x} - \underline{y}]$$

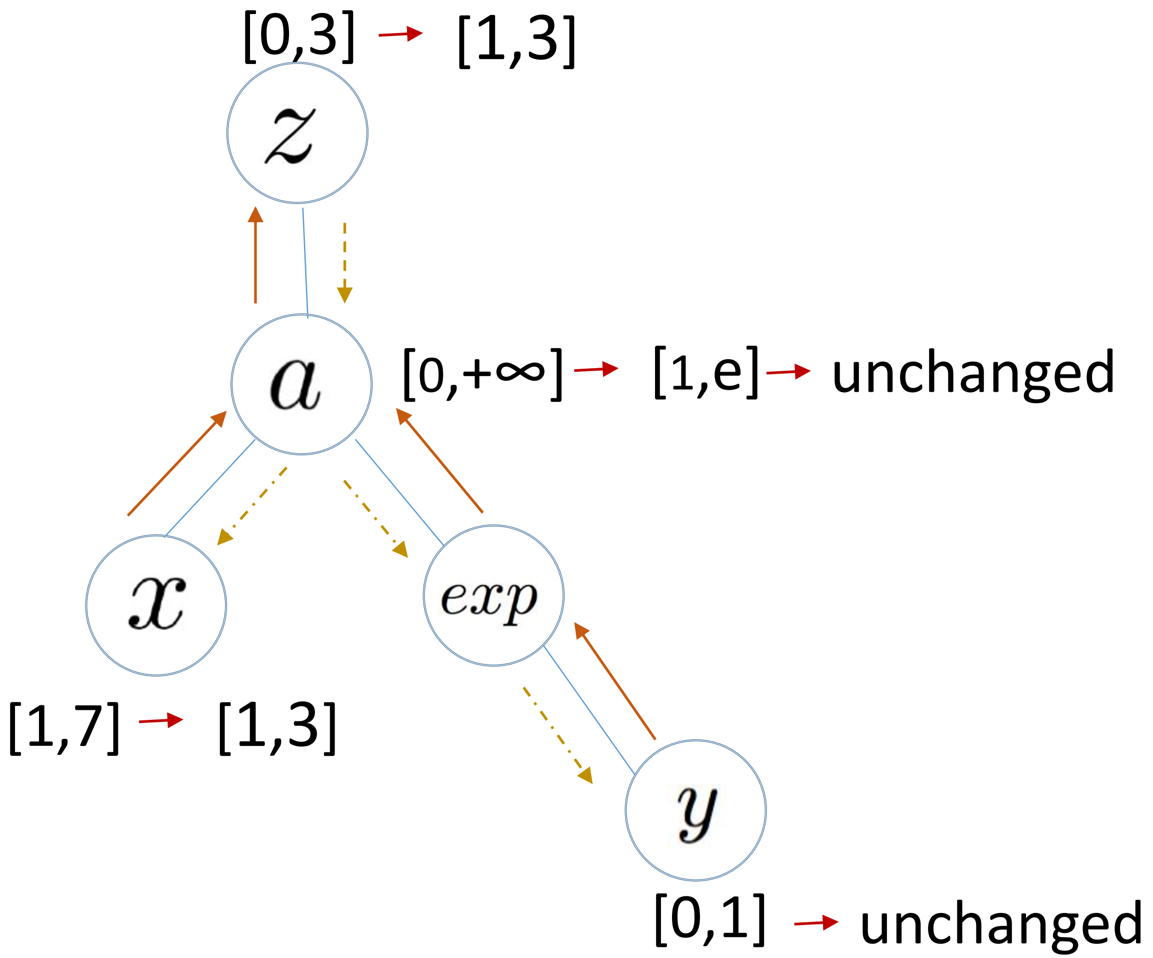
$$[x] \cdot [y] = [\min(\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\overline{y}), \max(\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\overline{y})$$

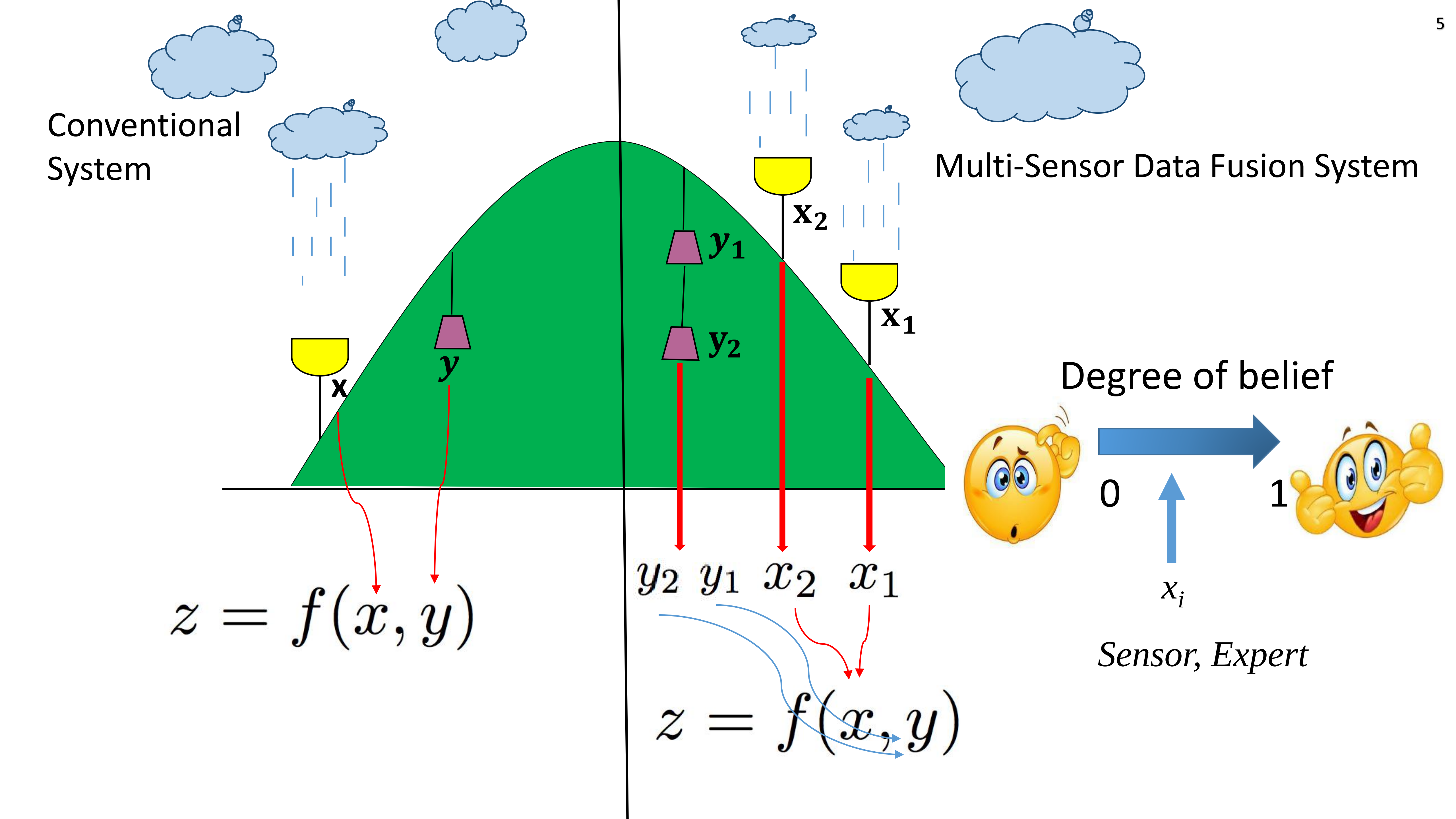
If the bounds of $[x]$ and $[y]$ are positive, we also have

$$[x]/[y] = [\underline{x}/\overline{y}, \overline{x}/\underline{y}].$$

Forward Backward Propagation

$$z = x \cdot e^{y} \rightarrow a$$

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x
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$z = f(x, y)$
y_2 y_1 x_2 x_1
$z = f(x, y)$
Degree of belief
0
1
x_i
Sensor, Expert

Dempster-Shafer Theory Definition

- We are interested in a **variable** ***x*** which takes values in a finite domain **Ω**, called *"frame of discernment"*

- A_i is a subset of **Ω**.

- We assign a **mass** $m(A_i) = m_i \in [0, 1]$ to a set A_i

- The summation of masses of all subsets is equal to 1.

Conventional System

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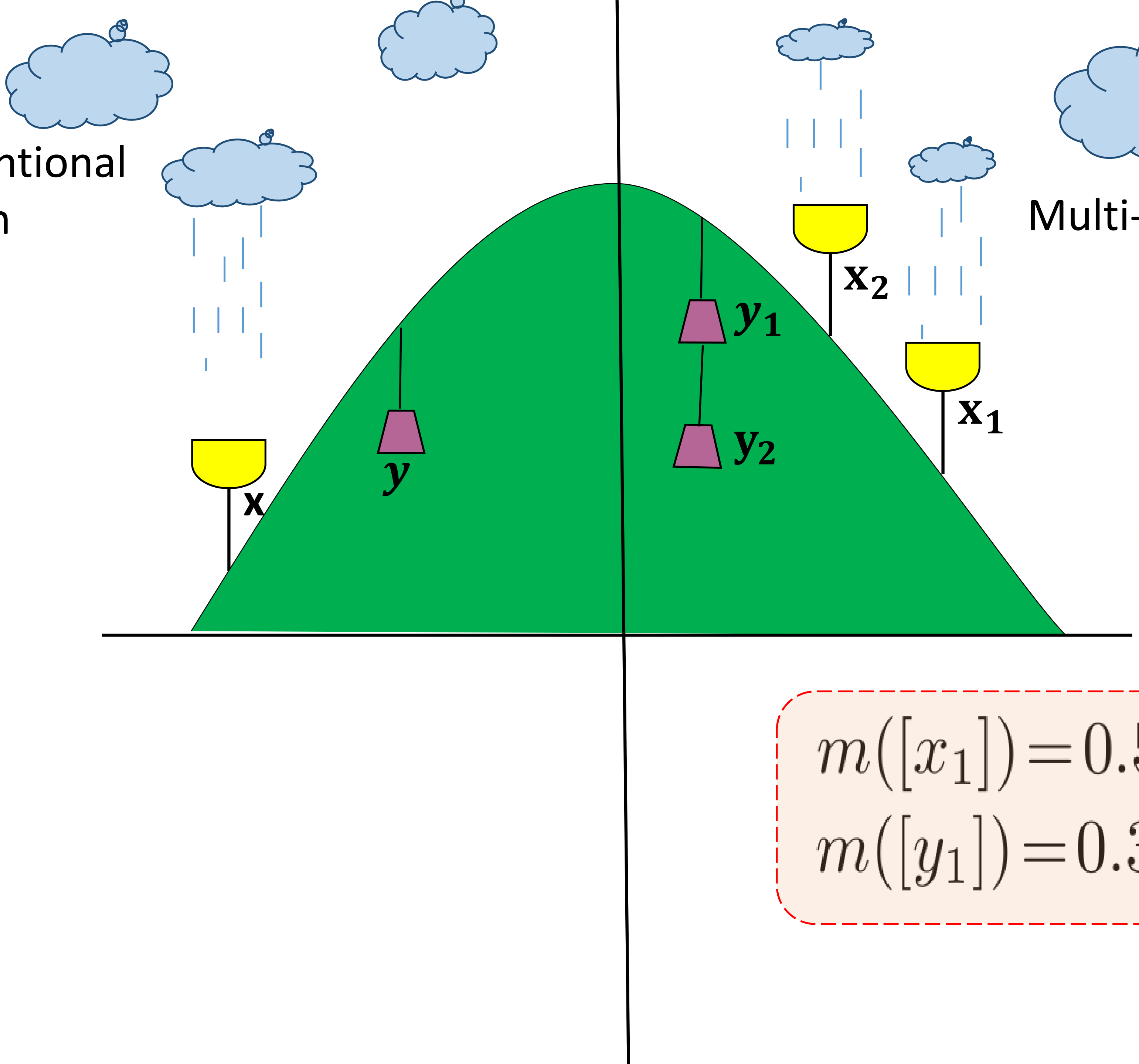

$$z = f(x, y)$$

$$m([x_1]) = 0.5, m([x_2]) = 0.5$$
$$m([y_1]) = 0.3, m([y_2]) = 0.7$$

Dempster-Shafer Theory Definition

- We are interested in a **variable *x*** which takes values in a finite domain **Ω**, called *"frame of discernment"*

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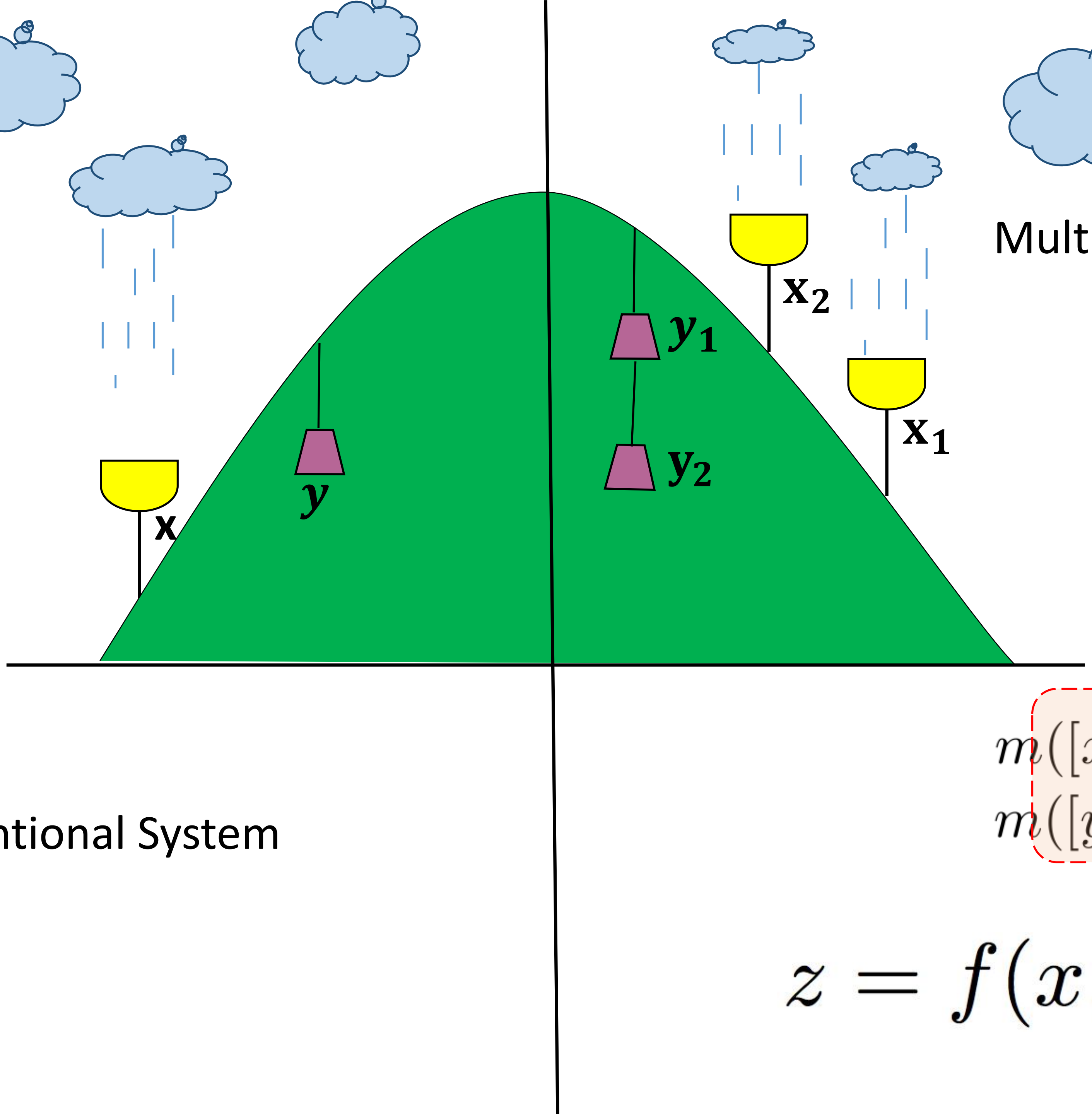
- The summation of masses of all subsets is equal to 1.

- ***Focal Set*** **,** $m(A_i) > 0$



Conventional System

Multi-Sensor Data Fusion System

Focal set

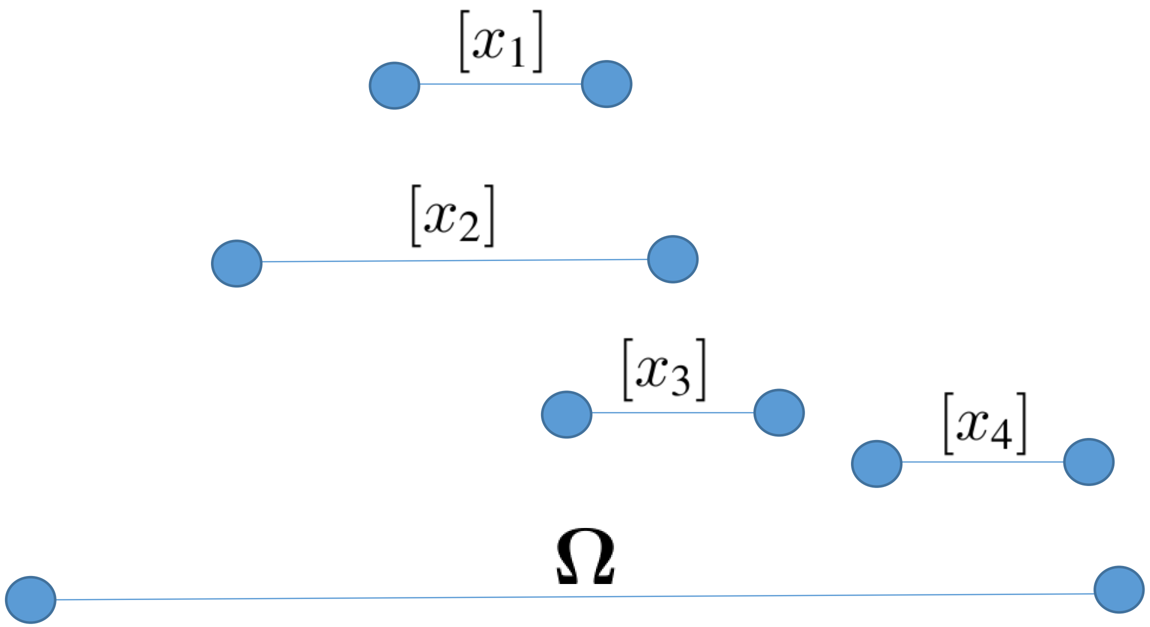
$$m([x_1]) = 0.5, m([x_2]) = 0.5$$
$$m([y_1]) = 0.3, m([y_2]) = 0.7$$

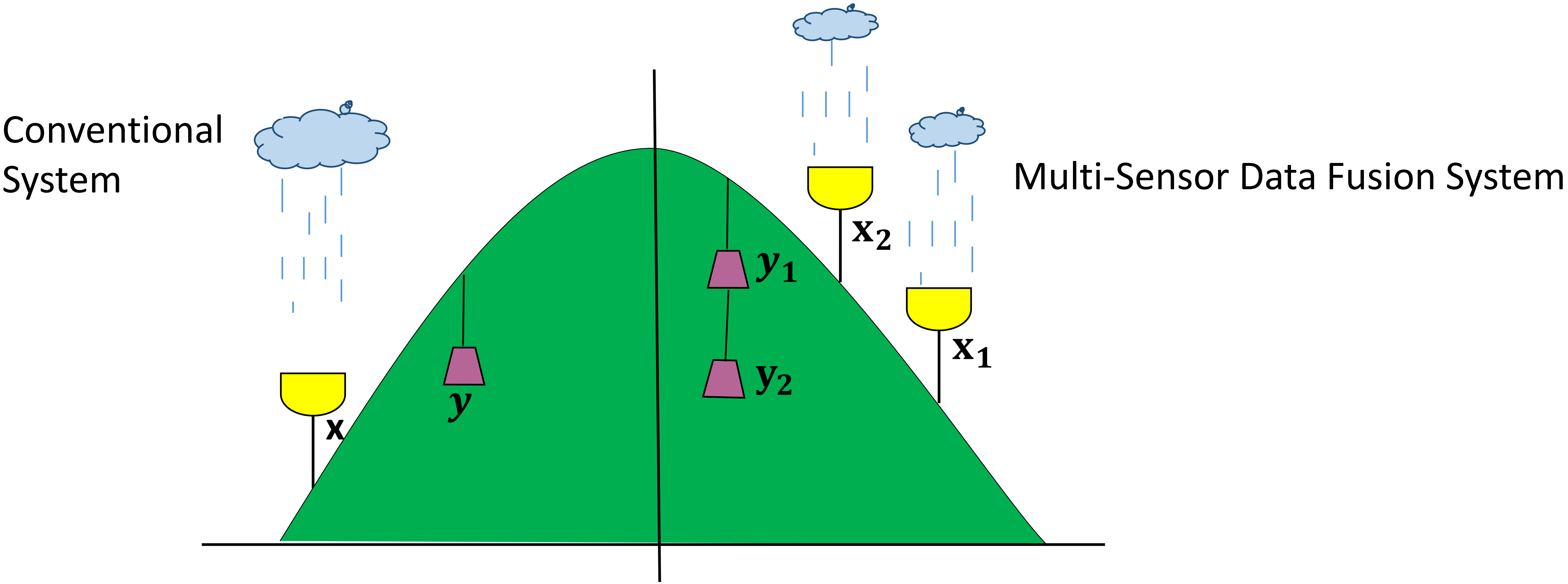
$$z = f(x, y)$$

Belief and Plausibility functions

$$bel(A) = \sum_{\{i|A_i \subseteq A\}} m_i$$

$$pl(A) = \sum_{\{i|A_i \cap A \neq \emptyset\}} m_i$$

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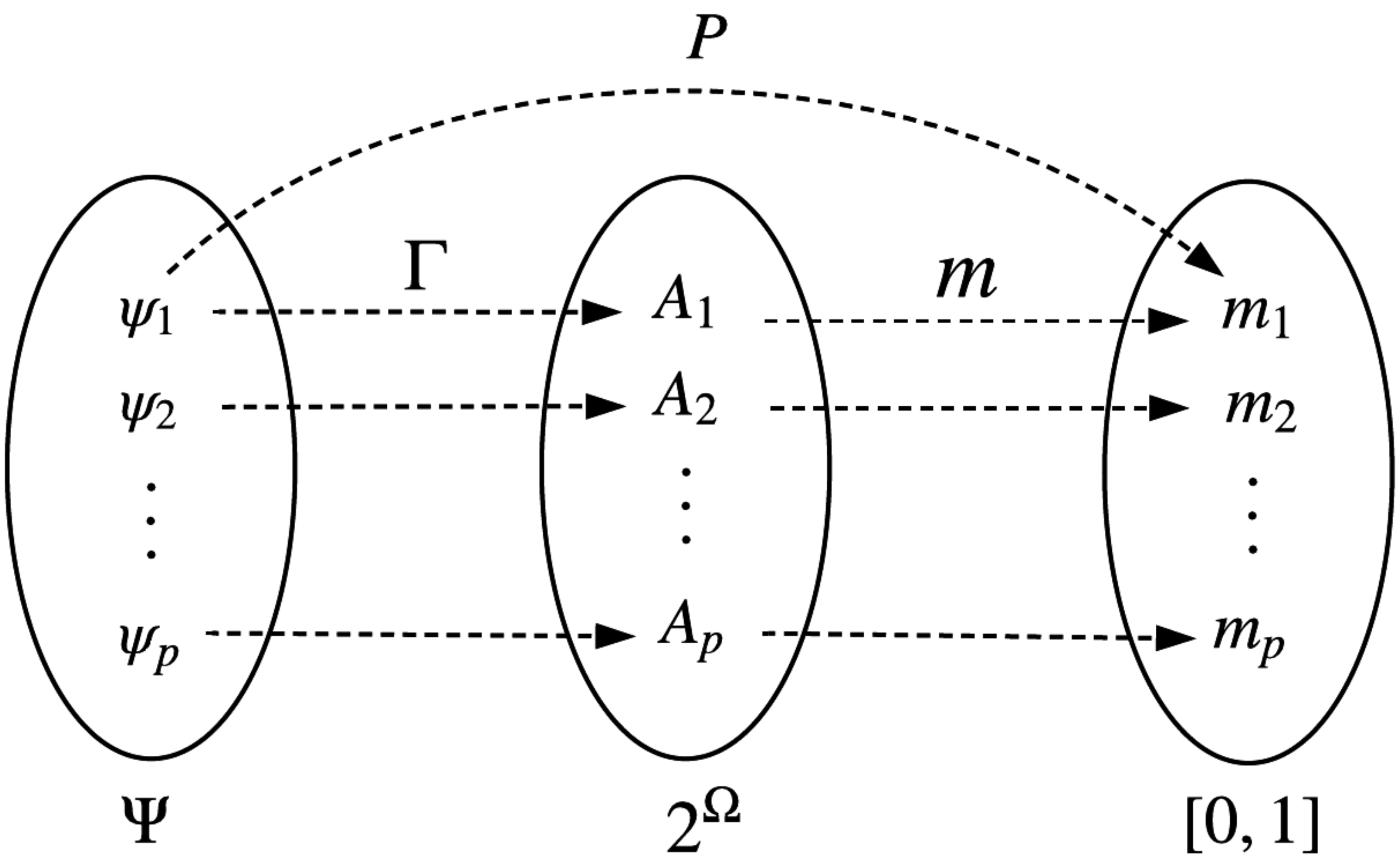
$$z = f(x, y)$$

Multivalued Mapping



Multivalued Mapping Conditions

$$z = f(x, y)$$

$$m^z([z]) = \sum_{\substack{\{i,j|[z]= \\ [\mathbf{f}]([x_i],[y_i]\}}} m^{x_i} \cdot m^{y_i}$$

Expectations

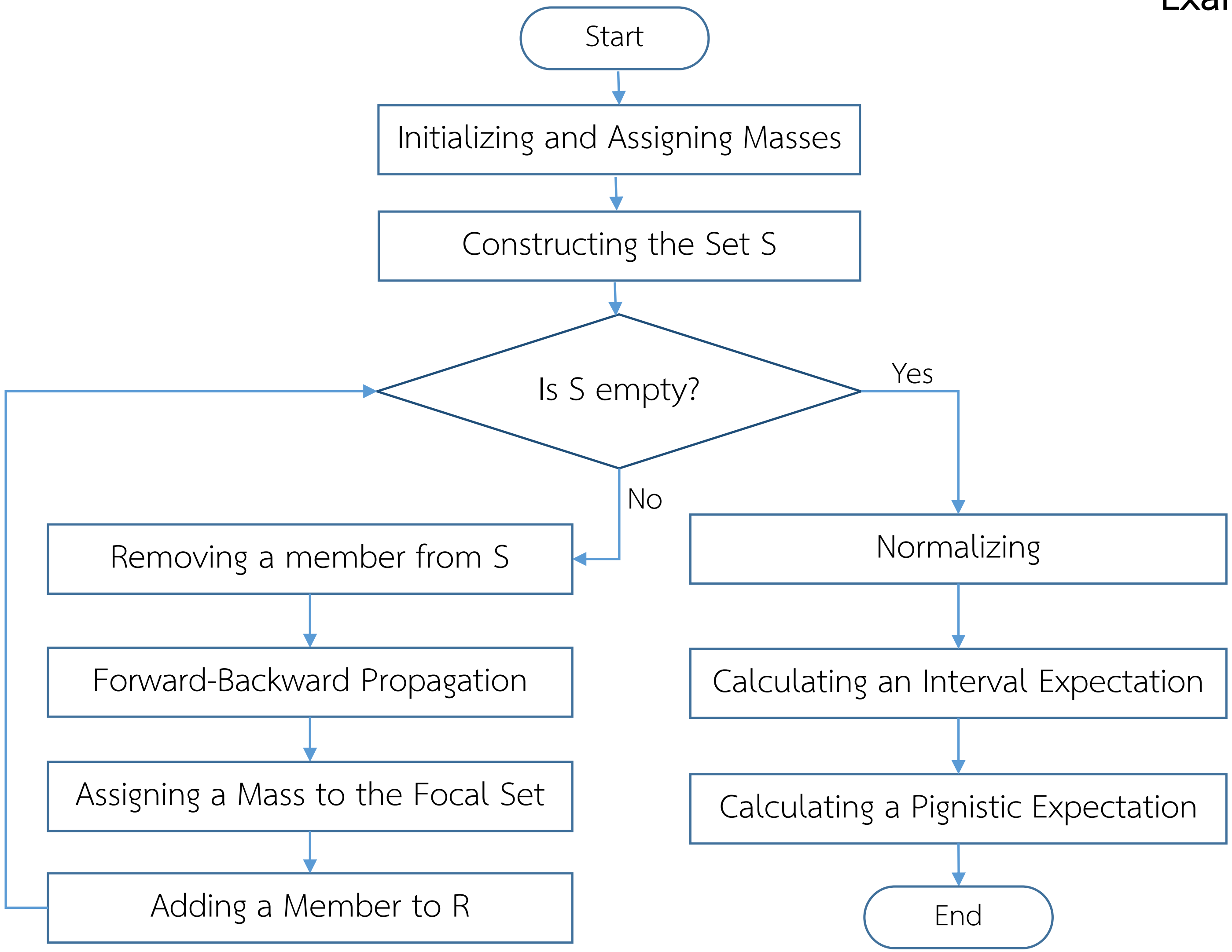
The interval expectation

$$[\mathbb{E}](m) = \sum_{j=1}^{p} m([z_j]) \cdot [z_j]$$

The pignistic expectation

$$\mathbb{E}(m) = \sum_{j=1}^{p} m([z_j]) \cdot c_j$$

Flowchart



Example : $z = f(x,y)$

$x_1 = [-3,3], x_2 = [-6,6], y_1 = [20,30], y_2 = [25,35]$

$m^x([-3,3]) = 0.5, m^x([-6,6]) = 0.5$

$m^y([20,30]) = 0.5, m^y([25,35]) = 0.5$

$S = \{ \quad , \quad , \quad , \quad \}$

$[z_1] = [27,29]$ → 0.33

$[z_2] = [24,29]$ and $m([z_2]) = 0.5 \cdot 0.5 = 0.25$ → 0.33

$[z_3] = \emptyset$ and $m([z_3]) = 0.5 \cdot 0.5 = 0.25$ → 0

$[z_4] = [24,26]$ and $m([z_4]) = 0.5 \cdot 0.5 = 0.25$ → 0.33

$R = \{ \quad , \quad , \quad , \quad \}$

$$m^z([z]) = \sum_{\substack{\{i,j \mid [z] = \\ [\mathbf{f}]([x_i],[y_i]\}}} m^{x_i} \cdot m^{y_i}$$

$[z] = [\mathbb{E}](m^z) = ([27,29] + [24,28] + [24,26]) \cdot 0.33 = [25,28]$

$z = \mathbb{E}(m^z) = (28 * 0.33) + (26.5 * 0.33) + (25 * 0.33) = 26.5$

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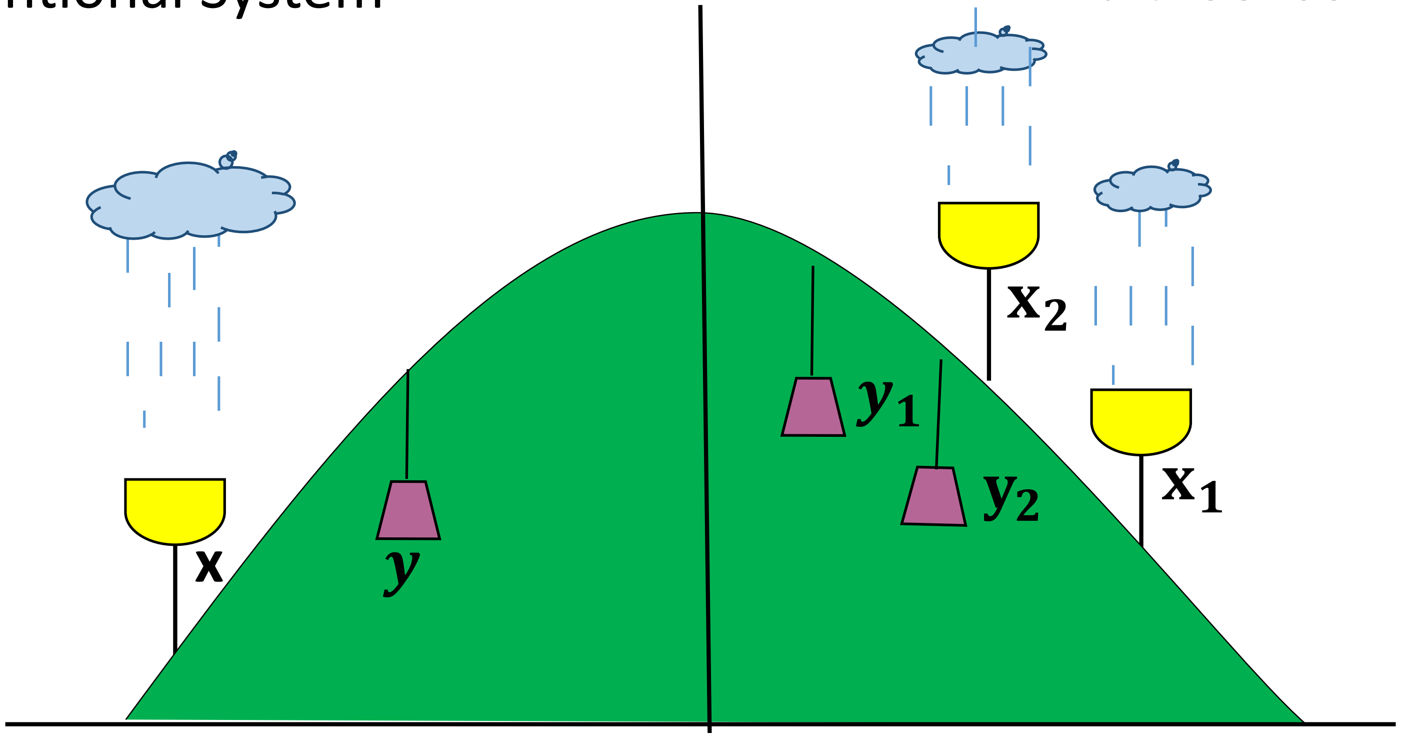

$$z = f(x, y)$$

*"We can use **Dempster-Shafer Theory** to **combine** our knowledge from **multiple sources** to **enhance accuracy**"*